THE EFFECT OF ECONOMIC SHOCKS UNDER DIFFERENT MONETARY POLICY PROCEDURES AND DIFFERENT ECONOMIC STRUCTURES

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THE EFFECT OF ECONOMIC SHOCKS UNDER DIFFERENT MONETARY POLICY PROCEDURES AND DIFFERENT ECONOMIC STRUCTURES

by

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To my Parents,
from whom I learned the fortitude to
achieve my goals.

To my husband Pedro,
whose support, confidence and love
have been with me every
minute and kept me going when
the road was rough.

To my children Maria Josefina and
Maria de Lourdes,
who have been my delight
in my toughest moments.

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THE EFFECTS OF ECONOMIC SHOCKS UNDER DIFFERENT MONETARY

POLICY PROCEDURES AND DIFFERENT ECONOMIC STRUCTURES

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This dissertation tries to contribute to the analysis of short run monetary policy, assuming it is used to attain a target level of income. To this effect the concept of Instrument-Indicators is introduced, representing monetary variables that are used as the link between the monetary sector and the income target, and that therefore serve to define a specific type of short run monetary policy. Each policy is defined according to a specific monetary variable that is used as the instrument to attain the final target. The purpose then is to analyze the effect of real and monetary sector shocks on income under each of the different policies, since these shocks may limit the ability of monetary policy to attain the desired income target. To do this an initial theoretical framework is specified, and then a set of

different assumptions is introduced yielding a set of different frameworks (models) under which our monetary policy analysis is performed. These frameworks or models are of two types: aggregated and disaggregated . For both types, the model's specification is summarized in the respective IS and LM equations. However, the monetary sector of the aggregated models does not disclose the elements that determine money supply, while the disaggregated models are characterized by a monetary sector that explicitely shows the sources and uses of the monetary base. For the former type of models we analyze the money supply and the interest rate policies, under economic structures that have the following unique characteristics:interest inelastic investment, interest elastic money supply, investment as a function of real expected interest rate and wealth effect in consumption. For the latter we study the security portfolio, unborrowed reserves, monetary base and interest rate policies, under economic structures that have the following salient features: interest inelastic demand for money, iinterest elastic borrowings and excess reserves functions and iinterest bearing demand deposits. Then for each model, and each alternative monetary policy, the effect of each type of shock on income is obtained in the respective reduced form equation. The consequences for the monetary variables that are not being used as instruments are also analyzed. The outcome of the analysis leads us to conclude that the superiority of a specific type of policy is determined not only by the type of instabilities or shocks that are more commonly present in the economy under study, but also by its structural characteristics. Therefore, an accurate specification of these characteristics is of greatest importance if monetary policy is to be successful. Furthermore, the need to revise the "rules" that have been established within a specific policy, as well as the framework behind such rules, arises whenever economic behavior is affected by current economic outcomes.

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CHAPTER 1

INSTRUMENT-INDICATORS OF MONETARY POLICY

1. Introduction

Monetary policy has always played a very important role in the economic performance of the United States, even greater than fiscal policy according to several studies dealing with the relative importance of each one.[1] Because of this, monetary policy is a crucial aspect of public policy, hence the relevance of research dealing with the existing mechanisms that lead to decision making in and implementation of monetary policy.

Within this context, the "indicator" problem deserves special attention. Choosing the most appropriate indicator of monetary policy has been the concern of many economists, as well as of the Federal

1. L. C. Andersen and J. Jordan, "Monetary and Fiscal Actions: A Test of Their Relative Importance in Economic Stabilization," in Review. Federal Reserve Bank of St. Louis, Nov. 1968. See also Frank De Leeuw and J. Kalchbrenner, "Monetary and Fiscal Actions: A Test of Their Relative Importance in Economic Stabilization. Comment", in Review. Fed. Res. Bank St. Louis, April 1969.

Reserve Board (hereinafter, FED) who, after having followed a strategy in which the indicator of monetary policy was interest rate performance, decided to start using unborrowed reserves as the indicator since October 1979.[2]

These indicators are part of the short run operating procedures by which the Fed has tried to attain monetary targets (i.e., money supply targets). During the 1970s, the targets were approached by setting and managing short run interest rates; however, since 1979 the Fed started using unborrowed reserves to this effect. In this sense, both interest rates and unborrowed reserves have played the role of instruments in the attainment of the monetary targets. Therefore, short-run monetary policy has tried to control money (i.e., the Fed's money strategy) by using and monitoring certain variables that can be managed through open market operations. These variables have been called "indicators" by some authors, and "instruments" by others. However, both characteristics are necessary when dealing with short-run policy management.

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2. See Stephen H. Axilord and David E. Lindsey, "Federal Reserve System Implementation of Monetary Policy: Analytical Foundations of the New Approach", American Economic Review. Papers and Proceedings. Vol. 71, No. 2, May 1981, pp. 246-252. See also: Lawrence H. Meyer, ed., Improving Money Stock Control. Problems, Solutions and Consequences. Federal Reserve Bank of St. Louis, 1983.

The purpose of this chapter is to define these so called "instruments" or "indicators" as part of monetary policy strategy, to describe the properties that the chosen variables should have in order to play their role as part of the overall policy procedure, to distinguish them from "monetary targets", and to set the principles under which alternative variables can be evaluated as candidates to play the "instrument" and "indicator" role.

2. Definition of Instrument-Indicators

The indicator problem has been approached in two different ways: first, as the problem of finding an indicator of monetary policy actions (the taxonomical type, according to Saving[3]) and, second, as the problem of finding an indicator of the effect of monetary policy actions in economic activity, that is, in economic targets such as output, employment, prices, etc.

Brunner and Meltzer[4] define indicators as the "measures of the direction in which monetary policy has changed in the recent past and/or of the effect of monetary policy on employment". Saving[5]

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3. Thomas Saving, "Monetary Policy Targets and Indicators", in <u>Journal of Political Economy</u>. Vol. 75, No.4, 1967, pp. 446-60.

4. Karl Brunner and Allan Meltzer, "The Nature of the Policy Problem", in <u>Targets and Indicators of Monetary Policy</u>. Brunner and Meltzer, eds., San Francisco Chandler Publishing Co, 1969, pp. 1-26.

perceives the indicator problem as that of "finding a variable or combination of variables that will best describe the effect that current monetary policy is having on economic activity". He further argues that "the importance of the problem arises from the fact that the choice of future policy is influenced by the policy maker's estimate of the effect of his current policy".

Therefore, we have, on the one hand, indicators as the variables showing the directions followed by the FED in the recent past and, on the other, indicators as signals about the effects of monetary policy in the economy. In the context of evaluating different policy alternatives and selecting the most appropriate course of action for the future, the distinction is important because a situation may arise in which the first indicator definition yields a different result from the second one. For example, an expansive monetary policy (e.g., an increase in unborrowed reserves) , may not have such expansionary effect due to several factors or responses, that may or may not be always present, and which are part of the trasmission mechanism of monetary policy. It is clear that under such circumstances, what monetary authorities ought to look for is an indicator that could show if policy actions are having an expansionary effect on economic activity , regardless of whether those actions were expansive or not.

5. Thomas Saving, op. cit. pp. 446-60.

Having a reliable indicator of the effects of policy actions on economic activity is necessary because it permits the consideration and evaluation of future policies; more important, it also implies the need for a variable that can be observed as often as policy decisions are made. Hence, the indicator problem is a short run problem: indicators inform policy makers about the performance of the economic goal variables that are their main concern, but are unobservable in the short run.

As Saving states, " the variables in which we are ultimately interested are observable with a considerable lag, so that by the time we find out what the effect of our policy has been, considerable damage may have been done. Hence, the choice of indicator does not imply that useful information is being discarded but only that all the information is not necessarily useful or available or both."[6]

Table 1-1 shows some of the properties that have been advanced by several authors concerned with the indicator problem. These can be classified as follows:

1. Controllability. As the Table indicates, some agree that it is desirable to define indicators as variables that are easy to control, which implies that they should be very sensitive to policy actions and hence highly manageable through changes

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6. Thomas Saving. op. cit., pp. 452-53.

Table 1-1. Desired Properties for Monetary Policy Indicators.

Author	Properties: Controllability	Effects on Economic Activity	Exogenous with respect to rest of Economy	Information Availability
Saving	Easily affected by monetary policy actions	Related to Target or Goal Variables. This relation at least qualitatively predictable	Exogenous changes (i.e., apart from monetary policy) should not affect it.	Easily observable with little or no delay
Brunner and Meltzer	Closely related to policy actions	Measure of the rate at which monetary policy is pushing the economy towards or away from goal variables	Be affected little by other policy variables and predetermined variables	Information available with minimum lag
Tobin	Easy to Control	Be links or chain of causation from instruments to economic goals		
Brainard		Linkage between monetary actions and economic activity		
Kamier and Schwartz		Indicate effect of recent policy on economy's moving towards broad economic objectives		Little information required
Hamburger	Subject to Fed's control	Be crucial in trasmitting effects of Fed in economy		
Davis		Effects on economic activity crucial and will depend on diverse sources of instability affecting economic system		
Havrilesky		Economic structure will determine which variables would perform best as indicators according to their trasmission links to goal variables		
Barnett		Prediction ability as measured by information content of indicator to estimate or predict the unobserved goal variables		

Sources:Thomas Saving (1967), Brunner and Meltzer (1967), Tobin(1969 and 1983), William Brainard (1969), Kamier and Schwartz (1969), Hamburger (1974), Richard Davis(1974), Thomas Havrilesky(1977) and William Barnett(1981).

in those actions (i.e., through open market operations, the discount rate, reserve requirements, etc).

2. Effects on Economic Activity. A second property, shared by most of the selected authors, refers to the role of indicators in the context of the economic system: they must have a definite influence in the variables that have been selected as goal variables for the economy as a whole. Taking these two properties together (i.e., "Controllability" and "Effects on Economic Activity"), indicators come to play the role of the instruments to attain the desired ultimate economic targets.

3. Endogeneity with respect to the rest of the economy. The third property, which complements the first one and is stressed by many authors, refers to the need of selecting variables that are affected insignificantly (or not at all) by policies or phenomena other than monetary policy actions. In other words, we need variables whose endogeneity with respect to the rest of the economy is minimum. This would reassure the controllability of indicators and minimize the possibility of changes that may be misleading , as signals of monetary policy effects in the economy in the process of using such indicators as the instruments to attain ultimate economic goals.

However, this may not be the case, since, as Brunner and Meltzer[7] argue:

> (Most of) the indicators of monetary policy usually mentioned by economists... are endogenous variables. As such, their position or rate of change at any time is the result of the joint interaction of the whole economic system and reflects more than the effect of current monetary policy. Fiscal policies and noncontrolled exogenous variables also influence the endogenous indicators.

While it may be difficult to find a monetary variable that can easily be affected by monetary policy actions and at the same time be completely unaffected by other economic phenomena , we can adopt a criterion in which the variables that are least exposed to these additional effects (or forces) will be considered best choices or , we may say, the variables that can more precisely (certainly) be affected basically by monetary policy actions, and that the interaction of monetary policy and uncontrolled forces affecting those variables, will render the best results in terms of the economic policy goals.

As Brunner and Meltzer recognize, "the relative size of policy and non policy influences will differ with the endogenous variable selected as indicator." [8]

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7. K. Brunner and A. Meltzer. "The Meaning of Monetary Indicators", in Monetary Process and Policy: A Symposium. G. Horwich, ed., Homewood, Ill., 1967, p. 190.

4. Information Availability. Finally, since indicators would represent a basis for future policy making, it would be convenient to obtain information on them with a minimum amount of delay. If ours is a short-run decision problem we need a short period of delay between the occurrence of important events and the information obtained.
5. Predictability Capacity. If we take all of the above properties together, we can say that a suitable indicator should be able to "predict" the effects of monetary policy in the economy. This property is really concerned with the second and third properties , we may say that it encloses them. Not only must the instrument- indicator have an impact on economic activity, but should also be able to predict such a relationship in a systematic (certain) way. It is certain that this ability will be affected by the third property. Although we may find many variables that can indicate some of the effects that monetary policy is having on economic activity, our indicator definition should incorporate the "instrumental" aspect which reduces the universe of variables to be used. In other words, the selection should be made among variables that can be used to attain the desired

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8. Ibid., p.191.

ultimate targets, which means that by observing their performance the targets' behavior can be monitored.

Finally, it is important to mention that due to the velocity of the recent changes in the financial system (i.e., the diversification of financial assets), there may be "possible" instruments or indicators that do not show changes, (and hence do not signal economic activity changes) under circumstances in which economic performance is being affected by the varying composition of financial assets (which is part of a monetary policy package). Therefore the ability of an indicator to show such a compositional change effect will affect most of the above properties, especially the predictability ability.

Therefore, we need to deal with instrument-indicators instead of instruments or indicators as two separate and distinct concepts since, as we have said, both properties are required in a variable to be used as pivot for monetary policy. This discussion leads us here to define an <u>instrument- indicator</u> (hereinafter I-I) as that monetary variable that can be manipulated and through which ultimate targets can also be controlled (this is the "instrumental" aspect), and that will serve as a feed-back of information about ultimate targets (this is the "indicator" aspect). In short, the properties that are required in an instrument- indicator of monetary policy are: its controllability by the monetary authorities(response to direct policy changes, both quantitative and qualitative such as asset diversification) , the possibility to predict its relationship with economic activity, especially with the economic goal variables, in a systematic way (hence the possibility to use it as pivot of monetary policy decisions), and the feasibility to obtain information about its behavior with little or no delay.

3. Difference Between Instrument-Indicators and Monetary Targets.

It is now necessary to distinguish between the concepts of monetary indicator and monetary target. Thomas Saving[9] emphasizes the necessity of differentiating between a target and an indicator. He states:

> Essentially the policy maker requires a separation of the change in his target variable into a policy effect and an exogenous effect. Since observations of the changes in the target variable yield only the total effect, some other variable or combination of variables is required to reflect the policy effect.

He then refers to such variable reflecting the policy effect as the "monetary policy indicator". Taking into account this distinction and the previous I-I definitions, we summarize the link between monetary policy actions and economic activity in Figure 1-1.

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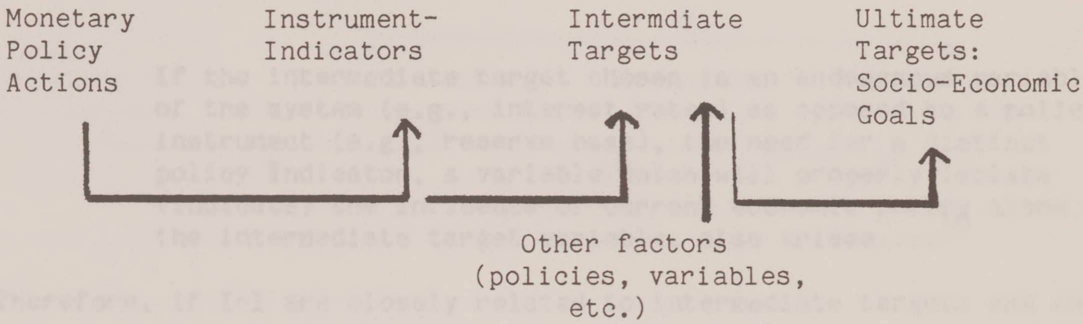
9. Thomas Saving, op.cit., p. 450.

Figure 1-1: Targets and Instrument-Indicators
in the Context of Monetary Policy



According to Figure 1-1, there is (or must be) a direct relationship between monetary policy actions and I-I; hence, I-I must be related to targets (i.e., "intermediate targets", as many authors call them), and these must carry on the effects of monetary policy to the macroeconomic milieu as reflected by the broad economic goals (i.e., "final or ultimate targets").

Targets represent the link between monetary policy actions and economic goals, although they are also affected by other factors and policies. On the other hand, I-I are the variables that the monetary authorities would easily (or at least, better) control, and that would relate policy actions to targets such that we may be able to control target performance by following the paths of I-I . However, if the relationship between targets and I-I were unstable, the role of I-I as "instruments" in the control of targets and would place serious problems on overall policy procedure.

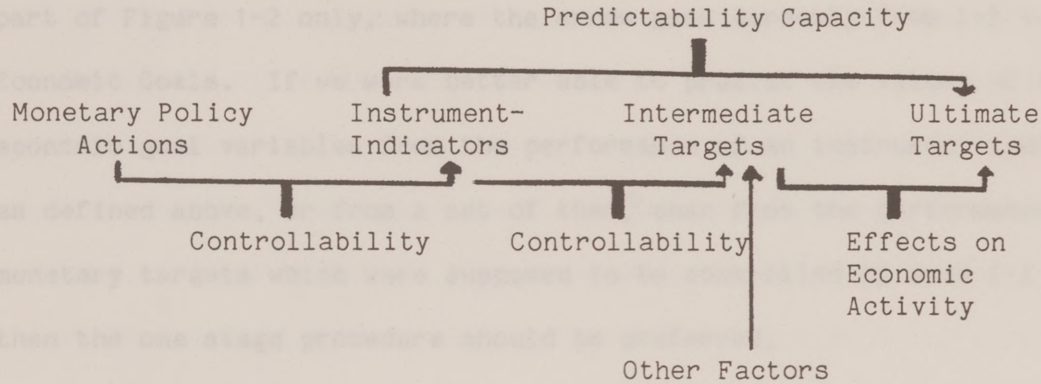
Thus, I-I should be managed (controlled) through monetary policy actions directly, and should also be capable of affecting targets in a determined (systematic) way:

> If the intermediate target chosen is an endogenous variable of the system (e.g., interest rates) as opposed to a policy instrument (e.g., reserve base), the need for a distinct policy indicator, a variable which will properly isolate (indicate) the influence of current economic policy alone on the intermediate target variable, also arises....[10]

Therefore, if I-I are closely related to intermediate targets and these are also related to economic goals, we should be able to predict economic performance by controlling of I-I (appealing to the predictability capacity of I-I). Notice that both I-I and intermediate targets are links between monetary policy actions and economic performance, but I-I , by our definition, are controllable (i.e., not affected by other factors) and also capable of affecting (controlling) ultimate targets in a systematic way (i.e., are the instruments of monetary policy). Figure 1-2 shows targets and I-I with the properties that are derived from the above definitions.

10. Robert Holbrook and Harold Shapiro, "The Choice of Optimal Intermediate Economic Targets", in American Economic Review. Papers and Proceedings. Vol. 60, No. 2, May 1970, p. 40.

Figure 1-2. Targets and Indicators: Desired Properties in the Context of Monetary Policy



The procedure described above corresponds to the so-called "intermediate- target strategy", which decomposes the monetary policy problem into two and where decisions are made separately at each of the two levels: socioeconomic goals are set to define intermediate targets; then, as intermediate targets have been established, I-I are to be managed to attain such targets. Alternatively, I-I could be viewed as the direct links between monetary policy actions and economic activity, by putting aside the monetary targets as described above. In such a case, intermediate targets would be omitted and the trasmission links between I-I and economic targets would be the relevant ones for policy effects. This would be an alternative to a two-stage or intermediate- targetting procedure, as suggested by Ralph Bryant:[11]

> An obvious alternative is to derive preferred time paths for policy instruments from the best feasible paths of the ultimate-target variables in a single-stage integrated

decision (for short, discretionary instrument adaptation) rather than interposing a two-stage process that pivots on an intermediate surrogate target.

In this case, monetary policy would be described by the upper part of Figure 1-2 only, where the arrow goes directly from I-I to Economic Goals. If we were better able to predict the values of the economic goal variables from the performance of an instrument-indicator as defined above, or from a set of them, than from the performance of monetary targets which were supposed to be controlled by such I-I , then the one stage procedure should be preferred.

Different arguments have been raised both in favor of and against an intermediate target approach, which is the one the Fed has followed since 1970. Specifically, such has been a "money strategy" since the money stock has been used as the intermediate target.

The advocates of the intermediate target (money) strategy base their proposals on diverse arguments such as: the need to control inflation by using a money target, the stability of the demand for money function and its low interest-elasticity which assigns an important role to monetary aggregates in controlling aggregate expenditure, the existence of long and variable lags in the effects of monetary policy and the existence of rational expectations. Advocates

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11. Ralph C. Bryant, <u>Controlling Money. The Federal Reserve and its Critics</u>. The Brookings Institution, Washington, D.C., 1983, p. 83.

also use arguments such as the view that such approach would use the available information more efficiently ,and the possibility to isolate monetary policy from the political process.[12]

On the other hand, some objections have been raised such as the impossibility of decomposing the economic system into two submodels such that a higher level one (where social goals relate to intermediate monetary targets) does not affect the lower level model (where monetary targets are used to define the adequate path that I-I should follow). It is also argued that focusing on intermediate targets may divert attention from the ultimate economic targets (i.e., social goals), and hence private agents may misperceive monetary policy. Finally, the possibility that monetary and fiscal policy would become more difficult to coordinate has also been used as an argument against intermediate targetting.[13]

Lastly, Davis[14] points out the structural limitations that may

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12. See discussion in Richard G. Davis, "Monetary Aggregates and the Use of Intermediate Targets in Monetary Policy", in New Monetary Control Procedures. Federal Reserve Staff Study, Vol. I, Board of Governors of the Federal Reserve System, Washington D.C., 1981, pp. 13-22. See also: Benjamin Friedman. "The Inefficiency of Short Run Monetary Targets for Monetary Policy", in Brookings Papers of Economic Activity. Brookings Institution, Washington D.C., No. 2, 1977, pp. 293-335.

13. See discussion in Ralph C. Bryant, op. cit., pp. 81-88.

14. Richard Davis, op. cit., pp. 38-42.

exist on the settings of intermediate targets and that could restrict the effectiveness of using such targets in combating inflation. For example, problems might arise due to the short-run effects of monetary policy on the real economy, before monetary policy has been able to affect inflation in the expected way.

However, it seems to us that the problem of choosing the most appropriate short-run operating guide has been mostly treated as a problem of money control rather that as a problem of controlling the ultimate target.[15] This view, when implemented, might prove to have dangerous effects if, with the purpose of keeping money supply in target, other monetary variables are freed to move having undesirable effects on ultimate targets.

The choice between an intermediate target and a single step approach to monetary approach would depend, as we have said, on the relationships that shape the underlying economic system and on the

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15. See for example: Axilord and Lindsey, op. cit., pp. 246-52; Albert Burger. "Money Stock Control", in Controlling Monetary Aggregates. The Implementation. Federal Reserve Bank of Boston, 1972, pp. 33-55;John H. Ciccolo, "Is Short-Run Monetary Control Feasible?", in Monetary Aggregates and Monetary Policy. Federal Reserve Bank of New York, 1974, pp. 82-89; Richard Davis. "Short-Run Targets for Open Market Operations." in Fed.Reserve Bank of New York, 1974, op. cit., pp. 40-59; J.M. Johannes and R. H. Rasche, "Can the Reserve Approach to Monetary Control Really Work?", in Journal of Money Credit and Banking. August 1981, pp. 298-313; and F. J. Levin, "Examination of The Money Stock Approach of Burger, Kalish and Bobb," Fed. Res. New York, op. cit., pp. 72-89;and Lawrence H. Meyer, ed., Improving Money Stock Control. Federal Reserve Bank of St. Louis, 1983.

stability of such relationships (e.g., those between instruments and final goal variables as opposed to those between intermediate targets and final goal variables).

The Fed's intermediate monetary target approach has been based on the belief that:

> The Federal Reserve can maintain a high (but by no means perfect) degree of control over operating variables (our "instrument- indicators). However, there is only a very loose relationship between these operating variables and goal variables. Given these circumstances , policy makers can find it useful to rely on intermediate target variables that are more closely related to economic goals than are the operating targets.[16]

However, some recent studies doubt the existence of certain and stable relationships between intermediate targets and ultimate economic goals. Higgins and Faust[17] analyze the velocity behavior of monetary aggregates (which measures the relation between GNP and the aggregate) in the case of U.S., and conclude:

> Recent erratic behavior of both M1-B and M2 has posed severe problems for monetary policy implementation. Although undoubtedly improving economic welfare by expanding the range of assets available to the public, recent financial innovations and changes in the legal and regulatory framework have distorted traditional money demand relationships, even

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16. Carl M. Gambs. "Federal Reserve Intermediate Targets: Money or The Monetary Base," in Issues in Monetary Policy:II. Federal Reserve Bank of Kansas City, March 1983, p. 15.

17. Bryon Higgins and Jon Faust, "Velocity Behavior of the New Monetary Aggregates", in Issues in Monetary Policy:II. Federal Reserve Bank of Kansas City, 1982, pp. 51-65.

> for the redefined monetary aggregates. As a result, the Federal Reserve's ability to achieve ultimate policy objectives by using monetary growth targets as policy guides may be impaired for the foreseeable future.

On the other hand, after analyzing the experiences with money growth targetting procedures in several industrial countries (Germany, Switzerland, Canada and U.K), Karen H. Johnson[18] concludes:

> Regulatory change, financial innovation, and shifts of many kinds all have the potential to distort the effects of a given target for money growth from those expected and intended. Foreign experience suggests that the ability to respond flexibly to disturbances of these kinds is an essential aspect of the implementation of targets for monetary aggregates.

Therefore, we cannot have definite conclusions about the superiority of a one-step or a two-step procedure until a specific economic structure (i.e., hypotheses about economic behavior) is assumed, and the corresponding relationships (i.e., between instrument-indicators and ultimate goals vs. intermediate targets and ultimate goals) are assessed empirically. However, the present study will be carried out assuming a one-step procedure from instrument-indicators to ultimate targets, under the economic structure that will be specified in Chapter 2.

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18. Karen H. Johnson, "Foreign Experience with Targets for Money Growth", in Federal Reserve Bulletin. Board of Governors of the Federal Reserve System, Washington D.C., Vol. 69, No. 10, Oct. 1983, pp. 745-754.

4.Importance of the Underlying Economic Structure.

The properties that have been described as desirable, given the role of I-I in the monetary policy process, are in fact trying to answer a specific question to the policy maker: Could one operating procedure, corresponding to a specific instrument-indicator, perform better than another in taking the economy closer to the ultimate targets, which is the ultimate goal of monetary policy? The answer to this question, will depend upon the economic structure under consideration, which will provide the possible outcomes of alternative monetary policy procedures.

Thomas Havrilesky[19] clearly states the importance of such a structure:

> The problem of choosing between alternative instruments is essentially a problem of choosing between competing hypotheses regarding the structure of the economy.

In the same way, several economists concerned with the management of monetary policy have approached it with specific objectives but basing their overall analyses on the belief that the underlying economic structure is crucial in the evaluation of any approach to monetary policy. Holbrook and Shapiro [20] evaluate money

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19. Thomas Havrilesky, "The Optimal Reaction Function: Confluence of the Instrument Problem and the Intermediate Target Problem", in Southern Economic Journal. Vol. 43, No. 3, Jan. 77, pp. 1288-1297.

supply vs. interest rates as monetary policy targets within a standard IS-LM constant price model, and conclude that different assumptions for the functional relationships would result in different recommendations as to the best variables to be used as targets. On the other hand, shifts in the behavioral relationships, (i.e, changes in the autonomous or stochastic part of the relationships) would also be significant in choosing one target rather than another. The relative stability of the commodity and monetary markets will determine which target would perform best.

Keran[21] stresses the importance of the structural framework assumptions in determining the best measure of monetary influence. He says that such measures would be determined by "the linkage between monetary variables and the rest of the economy", which itself depends "upon one's assumptions about economic behavior". He quotes as examples the Keynesian type of links which, if assumed, would lead to conclude that interest rates would be the best measures of monetary influences; on the other hand, if monetarist links are assumed, the money stock would be chosen.

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20. Robert Holbrook and Harold Shapiro, "The Choice of Optimal Intermediate Economic Targets", in American Economic Review Papers and Proceedings. Vol. 60, No. 2, May 1970, pp. 40-46.

21. M. W. Keran, "Neutralization of the Money Stock-Comment", in Review. Federal Reserve Bank of St. Louis, May 1970, pp. 12-14.

Richard Davis[22] also analyzes the performance of interest rates against money stock within an IS-LM framework, concluding that a money supply target would work badly in an economy subject to widely and unforeseen fluctuations in liquidity preference (i.e., money demand shifts), while interest rates will perform inadequately in an economy where the instability arises mainly from the nonfinancial sector (i.e., commodity market).

In an OECD study of U.S. Monetary Policy [23] there is also a reference to the instrument-indicator problem we are concerned with here. In this context, the study states the disadvantages of choosing as I-I variables that are endogenous, since they may give incorrect information about monetary policy effects in the ultimate targets. This assertion rests on an evaluation of alternative I-I according to specific economic structures. The conclusion is that an interest rate is a superior indicator when the typical source of uncertainities lies in the demand for money, while the money stock is a better indicator if the investment function is the main source of uncertainity.

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22. Richard Davis, "Implementing Open Market Policy with Monetary Aggregate Objectives", in Monetary Aggregates and Monetary Policy. Fed. Reserve Bank New York, 1974, pp. 7-19.

23. Organization for Economic Cooperation and Development, Monetary Policy in the United States. 1974.

Therefore, we can conclude that, first, any evaluation of alternative I-I and monetary procedures must depart from a set of assumptions about the economic structure (i.e., theoretical framework). Second, the usefulness of such an evaluation will depend upon the closeness of the specified structure to the economic reality for which policy recommendations are sought. Finally, the possibility that a chosen policy procedure could affect the underlying economic structure must be considered. This may well be the case when policy actions create certain kind of expectations and uncertainities , reactions that will themselves imply behavioral changes in the economic agents and hence in the structure of economic relations.

If the structural framework is specified, the key factors and relationships that determine the superiority of one monetary policy procedure with respect to another will emerge; furthermore, if such relationships could be assessed for a particular case economy, more specific conclusions might be obtained. This confirms the economics fact that there is no one solution for every case; there are at least as many possible solutions to a problem as many variations we can make to the general case.

After realizing the importance of the analytical framework in order to have any analysis of monetary policy procedures, as is our present concern, Chapter 2 is devoted to the specification of an analytical framework and its possible variations , while Chapters 3 and 4 analyze the effects of nonpolicy disturbances under alternative monetary policy schemes.

Summary.

1. In this chapter we have tried to show the importance of conciliating the concepts of instruments and indicators, which traditionally are dealt with as two different ideas within the analysis of short run monetary policy. We have defined I-I in the context of short run monetary policy, as the tools to be used in the process of attaining ultimate economic targets and the signals of their performance.

2. We have also defined the properties that I-I must fulfill in order to be useful in playing their role in the process of short-run monetary policy implementation. These are the following:

   a. Controllability. This term is used instead of "exogeneity" because the instrument-indicator should be at least closely related (in a predictable way) to monetary policy actions (e.g., open market operations), while it need not be necessarily the variable which is under the direct control of the authorities.

   b. Impact on Economic Activity. This property is a crucial one because it is embedded in the definition of the I-I . First of all, the definition of a Social Goal Function would establish a set of goals for economic variables which are the ones to be considered in evaluating the impacts of alternative variables to be used. Second, the impact that any variable has on economic activity is definite if we are to use it as an instrument and a sign of economic activity performance. This impact may be evaluated directly as the relationship between such variable and the economic goal variables (single-stage procedure), or in two steps (intermediate target procedure). In the latter case we would need to evaluate:

i. Target Control. Following the distinction between I-I and targets, and in view of the target role within the trasmission mechanism of monetary policy according to two-stage procedures, the evaluation of the relationship between I-I and targets is necessary since we expect it to be close and stable. This is tantamount to evaluating the control procedure in which monetary targets are pursued through monitoring and management of the I-I .

ii. Economic Activity Results. This implies an evaluation of the link between monetary (intermediate) targets and economic activity.

c. Endogeneity with respect to the rest of the economy. If an indicator of monetary policy is subject to influences which are not caused by monetary policy itself, it may not be an efficient instrument in that it does not give monetary authorities adequate information about policy effects, which they require to decide on future monetary policy actions. Such influences may distort the information pertaining to monetary policy. Therefore I-I must be affected little by phenomena other than monetary policy. The endogeneity of any monetary variable as well as its implications can only be assessed after a specific economic structure is specified.

d. Information Availability. This property refers to the easiness with which information about a variable is obtained: we must be able to obtain information about instrument-indicators with minimum delay.

e. Predictability Capacity. This refers to the indicator's ability of predicting the relevant economic variables (i.e., those in the social goal function.) In other words, are we acheiving our goals by keeping our I-I at the level that is dictated by our economic structure?

3. We have distinguished I-I from monetary targets and have recognized the possibility of either a two-step or a one-step short -run monetary policy strategy. In the first case, intermediate monetary targets, which link I-I with final targets, are established; in the latter case, these are not necessary.
4. The choice between a two-step and a one-step procedures will depend upon the structural characteristics of the economy under study, because these will determine the relationships between I-I and ultimate targets (i.e., types of trasmission mechanisms).
5. The choice of the most appropriate instrument-indicator within either a two-step or a one-step procedure will also depend upon the economic structure assumed. Different hypotheses about the economic structure may lead to different conclusions. We could establish a set of assumptions to conform a particular type of structure and get the respective outcomes from it; then, this structure may be changed to include or exclude specific cases and another set of conclusions will be obtained.

We conclude, first of all, that a theoretical framework incorporating the economic relationships that link the monetary sector with the rest of the economy must be specified. Chapter 2 will be devoted to this task. An initial theoretical framework is specified and then a set of new assumptions is introduced yielding a set of different frameworks under which our monetary policy analysis will be performed. This will allow us to capture the sensitivity of the monetary policy success to the structural characteristics of the economy. Then, using such frameworks, the effects of different shocks to the economy within alternative policy procedures (i.e., using different instrument-indicators), will be analyzed in Chapters 3 and 4. The superiority of a particular procedure will be defined in terms of the "Predictability capacity" of the corresponding I-I. This means the attainment of the desired target with the use of such I-I. Such predictability capacity may be sensitive to the economic structure assumed and to the types of shocks under analysis. Therefore, we expect to find important implications in this respect that may further qualify the adequacy of a specific monetary policy procedure.

We must finally mention that, since our analysis will be focused on the evaluation of instruments in the achievement of ultimate targets(one-step procedure), when analyzing a particular monetary policy procedure the respective I-I is assumed to fulfill the "controllability" property (i.e., authorities can keep it at a desired value). Although we will not try to evaluate this , we must note that such assumption will be more realistic the more disaggregated the instrument under consideration is, provided the instrument is a monetary aggregate. The possibility of analyzing instruments with different levels of aggregation will be evident in Chapter 2.

CHAPTER 2

ANALYTICAL FRAMEWORK

INTRODUCTION

This chapter will be devoted to the specification of the theoretical framework to be used as a basis for analyzing the effects of unpredicted economic changes (e.g., shocks, changes in behavioral relationships, etc.), under different short run monetary procedures.

The first part of the chapter contains a review of studies that have dealt, in one way or another, with the type of theoretical analysis we propose to do. The purpose and the model that these studies used, as well as their approach and results, will be described.

The second part of the chapter specifies the framework to be used in the present study. First, the elements and equations that determine aggregate demand (i.e., the real and monetary sectors), are described. Next, the monetary sector is disaggregated in order to capture the elements and mechanisms that determine money supply. This disaggregation of the monetary sector is necessary to determine the sources of money supply that are closest to Open Market Operations (hereinafter OMO), and that are therefore susceptible to be considered as instruments of short run monetary policy. Furthermore, this disaggregation will enable us to better understand the monetary factors that may either offset or contribute to the success of a chosen instrument in achieving the targets for which it is intended.

I. REVIEW OF THE LITERATURE

The first theoretical study that deals in depth with the instrument-choice problem for monetary policy is due to Poole.[24] His purpose is to determine the conditions under which interest rates should be used as instrument, and those under which the money stock is preferable. He assumes that authorities use monetary policy to stabilize income, and uses an IS-LM constant price model as the basis for his analysis. The authorities' loss function to be minimized is the following:

$$L = E(Y-Y_f)$$

where L = Loss

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24. William Poole. "Optimal Choice of Monetary Policy in a Simple Stochastic Macro Model", in Quarterly Journal of Economics. Vol. 84, May 1970, pp. 197-216.

Y = income

Y_f = full employment income

E = expected value operator

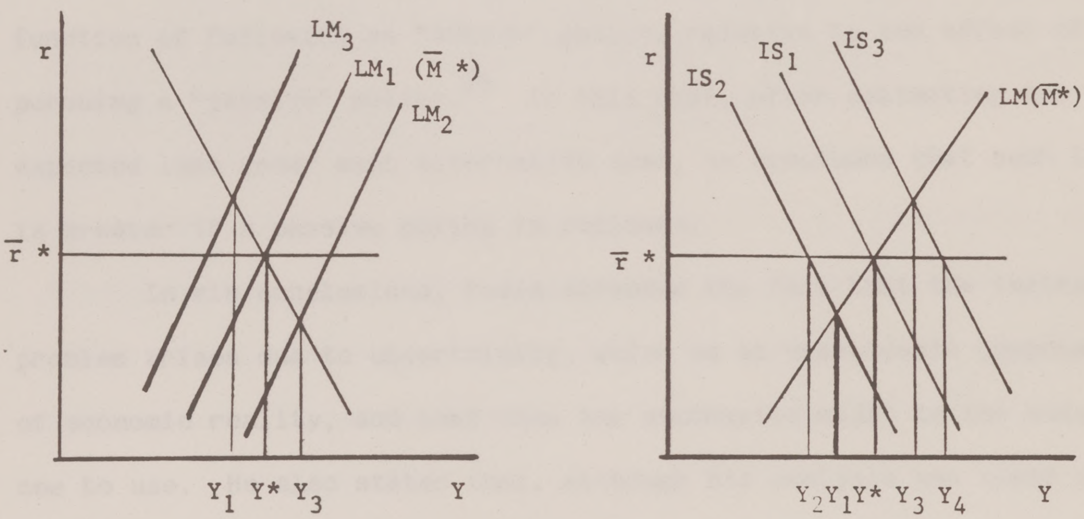
This loss function implies that income stabilization around full employment is the goal of monetary policy. The income variable he refers to is both nominal and real income, since he assumes a constant price level.

He then distinguishes between a deterministic and a stochastic model, and shows that in the former case the choice of instruments makes no difference, since either instrument will render the same solution. However, when stochastic disturbances are introduced into the behavioral equations, income becomes a random variable ; hence, the link between instruments and income is no longer perfectly known . Poole simulates random changes (i.e., shocks) in the real sector, as well as in the monetary sector and shows that if the real sector is relatively less stable, the money stock is the instrument that will minimize the loss function, while if the monetary sector shows less stability, interest rates render superior results. He also shows that the slopes of the IS and the LM affect the superiority of one instrument over another.

Figure 2-1 summarizes some of Poole's results as outlined above. The first part (A) shows that if the monetary sector is unstable (i.e., if LM fluctuates), the resulting equilibrium income

Figure 2-1

Poole's results under
IS and LM instability given
an interest rate policy (r*), or
a money supply policy (M*)



A. Monetary Sector
Unstable

B. Real Sector
Unstable

NOTES:

$\overline{r}^*$ = interest rate value if r is instrument

$\overline{M}^*$ = money supply value if M is instrument will always be Y_f under an interest rate instrument; on the other hand, if the instrument is the money stock, income will be fluctuating between Y1 and Y3 and so will interest rates. Part B of the diagram depicts the case when the real sector is the unstable one; we can see that if the instrument used is the interest rate, income will be fluctuating between Y2 and Y4, while taking money supply as the instrument will cause income to fluctuate between Y1 and Y3, which spans over a narrower range.

Poole then modifies the IS-LM model to introduce lagged effects (in the real sector or IS equation), and studies the effect on the loss function of following an "active" policy, relative to the effect of pursuing a "passive" policy.[25] In this case, after estimating the expected loss under each alternative case, he concludes that such loss is greater if a passive policy is followed.

In his conclusions, Poole stresses the fact that the instrument problem arises due to uncertainity, which is an unavoidable component of economic reality, and that thus the stochastic model is the adequate one to use. He also states that, although his analysis was based in the simplest model (since his goal was to show the importance and nature of the problem), it opens the way to perform the analysis in

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25. An active policy is defined as one that is adjusted period by period. The passive policy consists of setting a steady rate of growth of money or fixing interest rates, regardless of current economic conditions. See Poole, op. cit., p. 210.

more complex models. Finally, he comments on the uncertainity of the value of the model's parameters, something he did not consider in his analysis, but that could be the subject of further research on the topic.

Richard Davis[26] has also dealt with the instrument-choice problem in a study whose objectives are the following:

i) To consider and compare alternative variables to use as instruments of short-run monetary policy, taking into account the ability to control them through open market operations.

ii) To study the relationships of longer-term monetary targets and such instruments, in order to determine the degree of manageability of the targets by using a specific instrument.

In order to analyze the controllability of instruments, he decomposes each one to determine if it includes uncontrollable elements (from the authorities' point of view) that must be offset by OMO when the instrument is to be kept at a pre-established level.[27] He

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26. Richard Davis, "Short-run Targets for Open Market Operations," in Monetary Aggregates and Monetary Policy. Federal Reserve Bank of New York, 1974, pp. 40-59.

27. The potential instruments selected for analysis are variables within the transmission mechanism between OMO and the money stock (including the latter); that is, those variables that are monetary base sources, or the monetary base itself, or the money supply.

concludes that nonborrowed reserves and the nonborrowed base (adjusted for reserves required behind treasury deposits and free reserves), are the best choices as weekly monetary targets (instruments). His analysis also indicates that another group of instruments, including borrowings, total (adjusted) reserves, and the monetary base (also adjusted), could be used if the authorities had some knowledge of interest rate elasticities of the demand for excess reserves and borrowings.

The second part of his paper studies the relationship between instruments and targets, where the targets are also monetary variables. Davis suggests a policy procedure in which target values are specified on a monthly basis, from which the corresponding week-to-week values for the instruments will be specified.

Such procedure could be quantified by regressing the monthly values of the target in terms of the monthly average values of the instrument (weekly operational target) under analysis, of other lagged variables, and of seasonal dummies considered appropriate. He carries out this type of analysis using unborrowed reserves as the desired weekly instrument, and takes different monetary variables as the potential monthly targets. He then regresses the potential monthly targets as functions of the percentage change of unborrowed reserves, lagged unborrowed reserves, total reserves changes and some dummies. Finally, he does the same analysis using quarterly data. After analyzing the equations, he concludes that "there is no existing evidence to demonstrate the possibility of tight control over monetary and credit growth rates-even over quarterly average periods . . . Nevertheless, existing evidence does give reasonable grounds for hope that such control could in fact be possible over quarterly periods if midquarter corrections and the use of judgement can be brought in to substantial advantage."[28] Such conclusions, of course, are based on the lack of acceptable statistical results of the regression coefficients relating monthly targets and the instrument.

In the final section of his paper, Davis talks about the implications of having quantity (monetary) targets for the stability of the money market (i.e., interest rate behavior). The effects of a demand shift, given a quantity target, will be a change in money market conditions. Similarly, the effects of a demand shift, given an interest rate target, will be the accommodation of monetary aggregates. In his conclusions he argues that

> "Experience (in the U.S.) seems to indicate that the system can exert a very high degree of control over money market conditions if it chooses to disregard quantity considerations. By contrast, there is no experience to show what degree of control over monetary aggregates might be possible if such control were to be pursued exclusively and without regard to the effects on the money market. Similarly, there is no experience to show what the cost in terms of money market instability might be."[29]

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28. Richard Davis, op. cit., p. 49.

29. Richard Davis, op. cit., pp. 58-59.

However, Davis's study suggests that the adoption of explicit quantity targets would cause tolerable fluctuations in money market conditions if a "mixed strategy" is followed.[30] He also states that a policy to control the growth rate of an aggregate will still leave the month-to-month behavior of such aggregate subject to random shocks (noises), but that the control over longer periods of time (e.g., quarters) can be improved. As a summary of the role played by either quantities or interest rates as instruments he concludes:

> "If you wish to stabilize the price of any good, the amount you supply will reflect fluctuations in the demand schedule for the good; conversely, if you wish to stabilize the amount you supply, you must allow fluctuations in demand to be reflected in price fluctuations."[31]

Le Roy and Lindsey[32] explore the results obtained by Poole, which they justify mathematically in their appendix. They specify a simple static linear IS-LM structure, with independent, normally distributed errors and known coefficients, and analyze the superiority

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30. A "mixed strategy" refers to one in which both monetary aggregates and money market conditions play a role. That is, even if policy is defined so as to use a specific instrument (e.g., aggregate versus interest rate), the alternative instrument is not abandoned completely, but its behavior is used as a basis to decide whether to change the main instrument's value.

31. R. Davis, op. cit. p., 50.

32. Stephen F. Le Roy and David E. Lindsey, "Determining the Monetary Instrument: A Diagramatic Exposition", in American Economic Review, Dec. 1978, pp. 929-934.

of a money stock to an interest rate instrument if the objective were to minimize deviations of income from a target level (i.e., to minimize the loss function, equal to the variance of the error in the reduced form for income).

They simulate an unstable LM (and stable IS) situation as well as one in which it is the IS which is the unstable (while the LM remains stable). Their results are consistent with Poole's.[33] In the former case, the LM curve is responsible for interest rate changes and it is therefore preferable to use the interest rate as the policy instrument. In the latter, using the money stock as the instrument will render a smaller deviation of income from the target level.

They then simulate the case in which both IS and LM shift together (i.e., IS upwards and LM leftwards), and show that a money stock policy would result in too high an interest rate and a too low income level (relative to target levels), and that keeping an interest rate policy would cause both money and income to be too high. Under these circumstances they favor a combination policy in which,

> "The interest rate and money stock are maintained in the particular linear combination which minimizes the expected loss."[34]

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33. Poole, op. cit., 1970.

34. Le Roy and Lindsey, op. cit., p. 932.

Further simulation of IS and LM shifting together leads the authors to conclude that if the error variances of both curves were the same, a pure money stock policy would be preferred to a pure interest rate policy, regardless of the values of the slopes of the equations. They further state that if both, variances and slopes, were equal the optimum combination policy would coincide with a pure money stock policy.

Axilord and Lindsey[35] specify an analytical framework to analyze the operating procedures used in the United States before and after 1979. In other words, they study the reserve approach, used after 1979, in which unborrowed reserves are used as the instrument, and the federal funds rate approach, that uses such rate as the instrument. Their analysis incorporates the Fed's objective to attain a money supply target. The analytical framework implicit in their analysis is a stochastic model of the demand for and the supply of money, where the former depends upon income, prices and interest rates, whereas the latter is a function of unborrowed reserves and interest rates.

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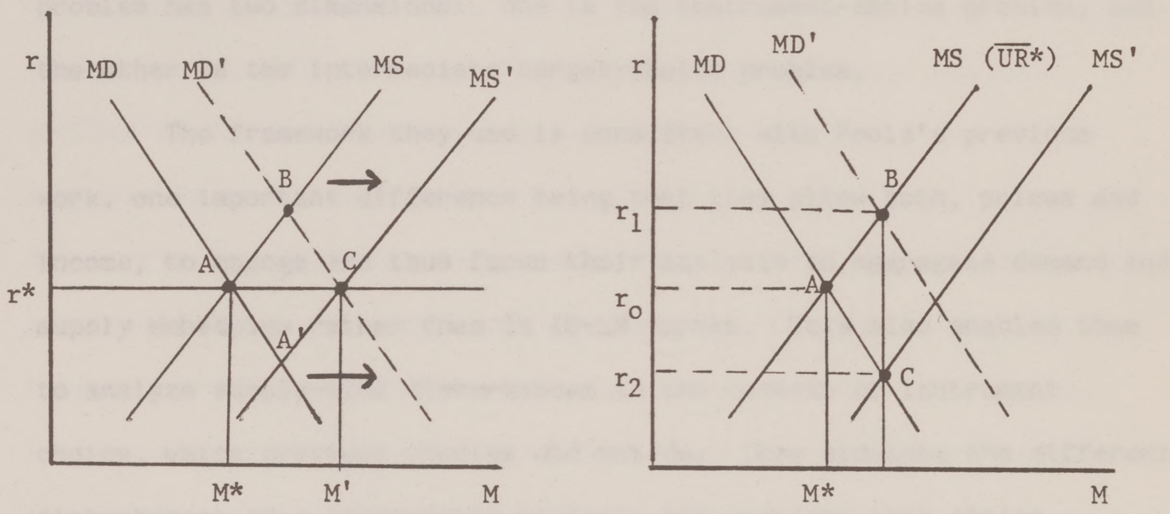
35. Stephen H. Axilord and David E. Lindsey. "Federal Reserve System Implementation of Monetary Policy: Analytical Foundations of the New Approach," in American Economic Review. Papers and Proceedings. Vol. 71, No. 2, May 1981, pp. 246-252.

Their analysis points out that within the reserve approach, either money supply or money demand shocks (i.e., shifts) would cause money supply and interest rates to change; however, the change in the interest rate will partially offset the effect in the money supply. The final result (deviation of money from target) would depend upon the interest elasticities of the supply and demand functions.

Within the federal funds rate approach, nonborrowed reserves would change to compensate demand shifts in order to keep the rate at the pre-assigned level, the money stock being therefore demand-determined. However, if there are money supply shifts, as unborrowed reserves are used to keep the pre-set interest rate, the money stock will return to its desired level. These results are shown in the diagrams of Figure 2-2, which show the results implied by the authors' analysis. Part A assumes an interest rate target. If there is a shift in the demand for money (to MD'), the money stock will become M_{1a}, since unborrowed reserves are increased in order to keep the interest rate at r*. Equilibrium first moves from point A to B and finally to C. On the other hand, if the money supply shifts to MS' autonomously, the interest rates will fall and unborrowed reserves are decreased in the process of restoring r*; hence M* is achieved. In this case, as Part A of Figure 2-2 shows, equilibrium moves from point A to A' and then back to A. Part B assumes an unborrowed reserves instrument (set at UR*). If there is a demand shift to MD', interest rates rise and so does the money supply, to r1 and M1b respectively

Figure 2-2

Effects of Money demand and Money Supply shifts on the Monetary Sector, under either an interest rate or an unborrowed reserves policy.



A. Interest Rate Policy

B. Unborrowed Reserves Policy

(notice that M1b is closer to M* than M1a). Equilibrium in this case shifts from A to B. Also, a money supply shift such as to MS' will cause interest rates to fall and the money stock to increase, to M1b and r2 respectively, equilibrium moving from A to C.

Sellon and Teigen[36] have also graphically formalized the analysis of short run monetary policy instruments, with the purpose of finding the circumstances under which interest rates are superior to reserve aggregates.

They focus on the U.S. case and so define (short-run) operating targets, intermediate targets and policy goals. Therefore, the choice problem has two dimensions: one is the instrument-choice problem, and the other is the intermediate target-choice problem.

The framework they use is consistent with Poole's previous work, one important difference being that they allow both, prices and income, to change and thus focus their analysis on aggregate demand and supply schedules rather than in IS-LM curves. This also enabled them to analyze supply-side disturbances in the context of instrument choice, which previous studies did not do. They simulate the different disturbances in a diagramatic analysis and consider both choice problems on each simulation. Their results are consistent with

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36. Sellon and Teigen, "The Choice of Short-Run Targets for Monetary Policy. Part I: A Theoretical Analysis," in Issues in Monetary Policy II. Federal Reserve Bank of Kansas City, March 1982, pp. 27-40.

previous studies. However, their analysis of supply side shock shows that in this case, the choice problem depends on the authorities' preference with respect to stabilizing inflation or real output. In the former case, a reserve aggregate is preferred as instrument, because if interest rates were chosen inflation would worsen. On the other hand, interest rates are the superior instrument in the latter case, since under such policy output tends to be restored.

Table 2-1 reproduces the authors' summary of their results for each type of disturbance.

Roper and Turnovsky[37] use an IS-LM stochastic model to take a closer look at the instrument-choice problem for income stabilization. However, they differ from previous studies in that they view the problem as that of selecting the most appropriate definition of the variable (monetary aggregate) to be used as the instrument. They define the aggregate as a weighted average of the monetary base plus government debt. The optimization procedure consists of finding the weights (of the monetary base and the government debt) that minimize income variance, subject to the IS-LM structure. Such IS-LM embodies the monetary aggregate definition and the government budget constraint. The optimization procedure yields the appropriate weights that the

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37. Don E. Roper and Stephen J. Turnovsky, "The Optimum Monetary Aggregate for Stabilization," in Quarterly Journal of Economics, Vol. 95, Sept. 1980, No. 2, pp. 331-356.

Table 2-1. Sellon and Teigen's Results: Choice of Short-Run Targets under Different Disturbances

Type of Disturbance	Appropriate Intermediate Target	Appropriate Operating Target
Spending (IS)	Monetary aggregate	Reserves aggregate
Portfolio (LM)	Interest rate	Interest rate
Money supply (LM)	Monetary aggregate or interest rate	Interest rate
Supply side	a) Monetary aggregate for inflation goal	a) Reserve aggregate for inflation goal
	b) Interest rate for real output goal	b) Interest rate for output goal

Source: Sellon and Teigen, "The Choice of Short Run Targets for Monetary Policy. Part I: A Theoretical Analysis", in Issues in Monetary Policy II. Federal Reserve Bank of Kansas City, March 1982, p.39.

optimal monetary aggregate should have under different disturbance assumptions. (i.e., it yields the optimal value of the weights, in terms of the variances of the disturbances of each market). The authors analyze special cases, such as that in which the disturbance in the goods market is zero. In this case, the solution is the same as in previous studies, namely that the optimal policy is to stabilize government debt, which is equivalent to stabilizing interest rates. Within their model and its assumptions, the authors conclude that introducing government debt as part of the monetary aggregate improves the possibility of stabilizing income. The only exception is the presence of an interest inelastic demand for money together with no disturbance in the demand for deposits (i.e., vertical LM), in which case the optimal aggregate equals deposits only, since the optimum solution renders a weight equal to zero for government debt. This result is expected under a situation in which the LM has become the sole determinant of income, such as an infinitely inelastic LM , and where the objective is to minimize income variance.

Ralph C. Bryant[38] analyzes alternative short-run operating strategies using a theoretical model for the money market. His framework is based on the balance sheet of the U.S. monetary

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38. Ralph C. Bryant, Controlling Money. The Federal Reserve and its Critics. The Brookings Institution, Washington D.C., 1983, Chapters 2 and 3, pp. 3-33.

authorities and includes the different elements that participate in determining the money supply.

He defines the alternative short-run procedures, each one corresponding to a particular instrument, as "policy regimes", and his analysis covers five different regimes: portfolio (government securities), base (monetary base), total reserves, unborrowed reserves, and funds rate. He then analyzes the effects of different types of disturbances, for each particular regime, within a policy procedure that intends to control the money stock (i.e., to achieve and hold a money supply target). That is, he evaluates the effects of a disturbance on money stock changes. He simulates disturbances occurring in the supply side of the money market, the demand side, and the real sector. Bryant concludes that the funds rate regime is the most appropriate for situations in which disturbances come from the supply side of the money market and also for disturbances on the demand side that are due to changes in assets' preferences. However, the funds rate regime proves inferior when he simulates aggregate demand disturbances originating in the economy's real sector, in which case the total reserves regime performs best. In his conclusions about the choice between interest rate vis-à-vis a monetary aggregate as instrument, he argues that, "the key point, however, is that neither a price-stabilization nor a quantity stabilization presumption is the appropriate guideline for monetary policy in all circumstances."[39]

Although his analysis only covers each disturbance case separately, he stresses the importance of realizing that in real life such shocks may occur simultaneously, giving the monetary authorities a very hard and uncertain environment to work in. Table 2-2 summarizes his results for each type of regime he considered.

Bryant's approach is based on the current structure of U.S. monetary policy, namely that a money stock target is specified as a link between instruments and ultimate targets. However, he does not believe that such a two-stage procedure (i.e., intermediate targetting procedure) is the most adequate if what authorities desire is to attain ultimate targets.

Finally, although he does not study international issues in the book we are referring to, he talks about their importance in the context of monetary policy. He contends that monetary interdependence between countries is a factor that makes monetary policy more difficult and its results more uncertain.

Tobin[40] has also emphasized the importance of economic shocks in the context of monetary policy procedures, stressing the fact that the optimum short run strategy (i.e., the instrumental choice), will

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39. Ralph Bryant, op. cit., p. 110.

40. James Tobin, "Monetary Policy: Rules, Targets and Shocks", in Journal of Money Credit and Banking, vol. 15, No. 4, Nov. 1983, pp. 506-518.

Table 2-2. R. Bryant's Results: Performance of Alternative Operating Regimes in Directing Non-Policy Disturbances to Least Troublesome Destinations

Operating procedure / Type of Disburbance	Portfolio regime	Base regime	Total reserves regime	Unborrowed reserves regime	Funds-Rate regime
Supply-side of monetary sector					
Float	F	A	A	A	A
Government deposit balance	F	A	A	A	A
Excess reserves	B	D	F	C	A
Required reserves	B	D	F	C	A
Discount window borrowing	D	A	A	F	A
Demand side of monetary sector					
Shift in deposit demand	B	D	F	C	A
Shift in currency demand	D	F	A	A	A
External to financial markets	C	B	A	D	F

NOTE: For a particular non-policy disturbance identified in a row of the table, each regime is ranked relative to the others.

A indicates the preferred regime for that disturbance, due to least troublesome consequences for Fed's objectives.
B, C, D indicate respectively the second, third, and fourth best regimes.
F indicates the regime under which the disturbance has the most troublesom effects.
Table should be read horizontally, one row at a time.

Source: Ralph Bryant, Controlling Money (1983).

depend "on your model of the financial and economic system, and on your objective and priorities".

He further argues that monetary policy should not only be concerned with nominal objectives since it also affects real variables such as the real interest rate, the real exchange rate, savings, investment, etc.[41] Monetary policy affects inflation, and inflationary expectations, which in turn affect real rates of return in currency and other assets with fixed nominal interest rate, and thus, the whole structure of asset prices and returns. In other words, he argues that "monetary authorities should not, indeed cannot, escape responsibility for real macroeconomic outcomes".[42]

Therefore, Tobin's position is that monetary policy must be concerned with both nominal and real macroeconomic variables. On the other hand, he reviews Poole's analysis of economic shocks and the convinience of pegging money versus interest rates, as well as a possible money supply-interest rate rule, as a form of combination policy. In this context, he argues that the LM curve has become steeper in the U.S. case as a result of monetary reforms (e.g., the deregulation of interest rates) and also due to the recent monetarist policy of focusing in money supply targets. According to him, such

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41. Ibid., p.509.

42. Ibid, p. 511.

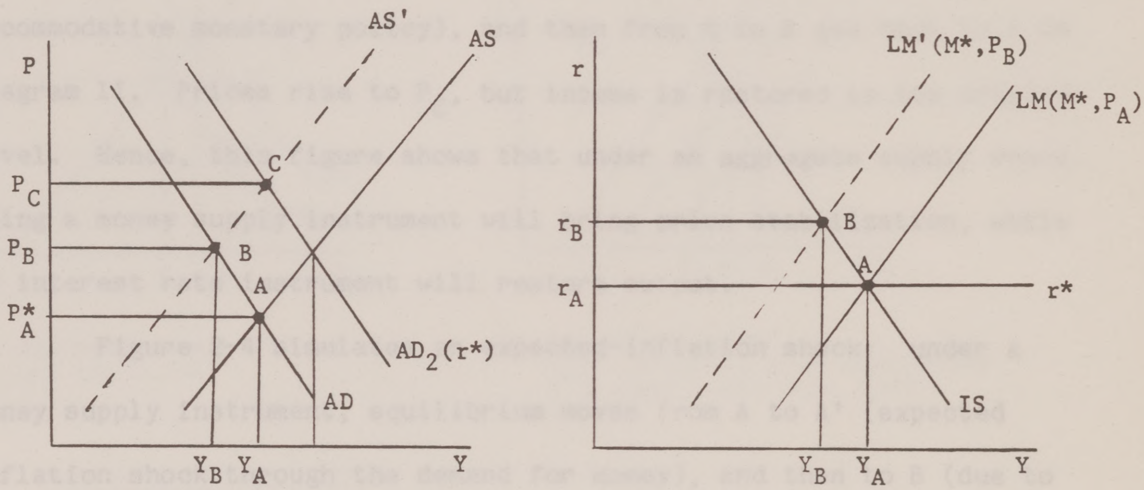
monetarist policy was established assuming a stable demand for money function, and the relative low probability of financial shocks as compared to real shocks; however, he admits that recent evidence denies the validity of such propositions.

Tobin stresses that the prescriptions arising from a new framework enclosing a volatile demand for money (financial shocks) as well as a steeper LM (due to monetary reforms) will be quite different from prescriptions that arise amidst optimum circumstances in which financial shocks would be greatly limited and the money market highly regulated.

He then analyzes the cases when price shocks occur in the supply side of the economy (i.e., aggregate supply shifts), as well as when there are expected inflation shocks (that cause a downward shift in the demand for money for given real interest rates). In the first case, the shock causes a fall in output and a price rise, which make Tobin conclude that if the authorities' main concern is inflation, a monetarist policy should be followed (i.e., fixing the money supply at a pre-established level), while if their main concern is to attain full employment income, an accommodative monetary policy (i.e., pegging interest rates) should be pursued (since real money supply fell, money supply must be increased). In the latter case, the effect of the shock is expansionary since the demand for money falls, in which case Tobin's analysis suggests that monetary policy be accommodative (i.e., pegging the interest rates that prevailed before the shock), increasing the

Figure 2-3

Aggregate Supply Shock
Interpretation of Tobin's Analysis



A. Aggregate Demand-
Aggregate Supply
Equilibrium

B. Real and Monetary
Sectors' equilibrium

NOTES:

AD = aggregate demand
AS = aggregate supply
AS' = aggregate supply after shock
r^* = interest rate value when interest rate policy
M^* = monetary supply value when monetary supply policy.

money supply when money demand has increased and reducing money supply when demand fell. Figures 2-3 and 2-4 try to give a diagrammatic interpretation of Tobin's analysis of these shocks.

Figure 2-3 simulates the aggregate supply shock. If money supply is the instrument, equilibrium moves from A to B, in both diagrams I and II. Prices rise from P_A to P_B and income falls. On the other hand, if the instrument is the interest rate, equilibrium moves from A to C in diagram I (since aggregate demand shifts with the accommodative monetary policy), and then from A to B and back to A in diagram II. Prices rise to P_C, but income is restored to its original level. Hence, this figure shows that under an aggregate supply shock, using a money supply instrument will bring price stabilization, while an interest rate instrument will restore output.

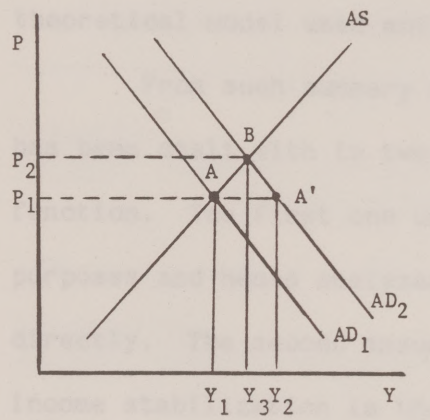
Figure 2-4 simulates an expected-inflation shock: under a money supply instrument, equilibrium moves from A to A' (expected inflation shock through the demand for money), and then to B (due to price rise), but under an interest rate instrument it shifts from A to B(as in the initial case) and back to A (due to the accommodative monetary policy).

It is clear from this diagramatic analysis that under such type of shock, an interest rate instrument would be more appropriate, regardless of whether the ultimate policy goal is to control inflation or the attainment of full employment income. Later in his paper, Tobin mentions the need to seek variables that would indicate or predict

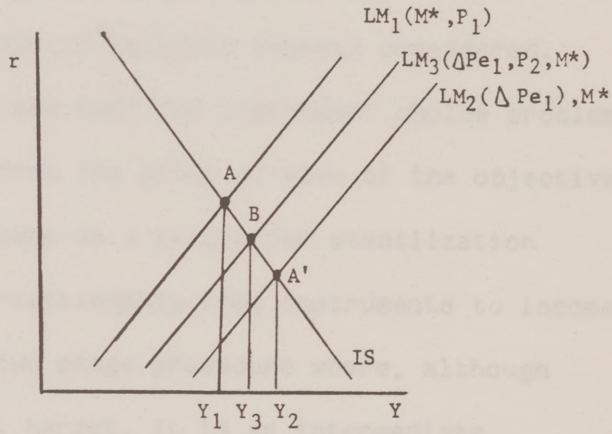
Figure 2-4

Expected-Inflation Shock:
Interpretation of Tobin's Results

A. Under Money Supply Policy:

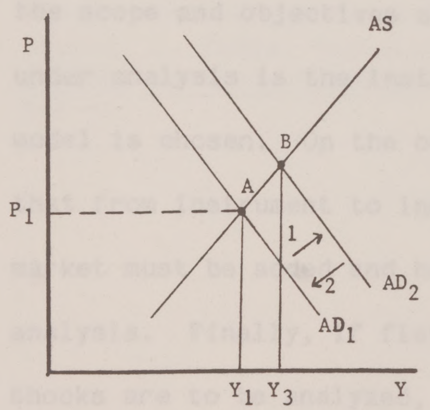


I. Aggregate demand (AD) and supply (AS)

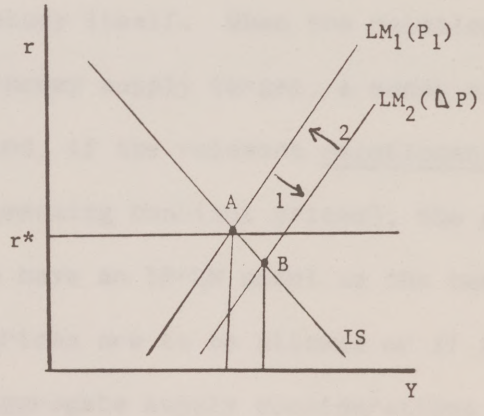


II. IS-LM Equilibrium:
ΔPe_1 = change in price expectations
M^* = money supply level fixed.

B. Under Interest Rate Policy:



I.



II.

types of shocks affecting the economy. He ends his discussion suggesting a multistage procedure for monetary policy implementation, within which instruments will play an important role. Such procedure is described in Table 2-3.

Table 2-4 summarizes the different studies described above, classifying them uniformly according to their main focus, the theoretical model used and the type of analysis (cases) considered.

From such summary we can see that the instrument choice problem has been dealt with in two ways from the point of view of the objective function. The first one uses income as a target for stabilization purposes and hence analyzes the relationship from instruments to income directly. The second assumes a two stage procedure where, although income stabilization is the final target, it is an intermediate monetary target which serves as the focus for choosing the most appropriate instrument.

The studies also vary with respect to the analytical frameowrk used. However, the definition of such framework is closely linked to the scope and objectives of the study itself. When the relationship under analysis is the instrument-money supply target, a money market model is chosen. On the other hand, if the relevant relationship is that from instrument to income (assuming constant prices), the goods market must be added and hence we have an IS-LM model as the basis for analysis. Finally, if flexible prices are to be allowed or if price shocks are to be analyzed, then aggregate supply considerations must be

Table 2-3. Tobin's Multistage Procedure for Monetary Policy

Ultimate
Target: (nominal GNP growth)
[2 years or more]

↓

Intermediate
Target: nominal GNP growth [1 year]

↓

Operating
Target: (rules) money stock, bank reserves, and
short-term interest rates [quarterly]

Source: J. Tobin, "Monetary Policy: Rules, Targets, and Shocks", in Journal of Money Credit, and Banking. Vol.15, No.4, Nov. 1983, pp. 516-17.

Table 2-4. The Instrument-Choice in Monetary Policy: Summary of Reviewed Theoretical Studies

Author	Key-Relationship Studied	Theoretical Framework	Type of Analysis
Poole (1974)	Instrument-income target	Stochastic, fix-price IS-LM	IS and LM shocks
Davis (1974)	a) Open Market Operations-Instruments and b) Instrument-Intermediate target	a) Decomposition of instruments b) Intermediate targets' reduced forms, in terms of instrument	a) Controllability of components b) Goodness of fit of reduced forms, egs
LeRoy & Lindsay (1978)	Instrument-Income target	Diagramatic stochastic IS-LM constant price	IS, LM and IS+LM shocks
Lindsay (1981)	Instrument-money supply target	Money-supply-money demand model	Money supply & money demand shocks
Sellon & Teigen (1982)	Instrument-Intermediate target-income target	Diagramatic Model: aggregate demand, aggregate supply, IS-LM, money supply-money demand, demand for reserves-supply of reserves	IS, LM, and aggregate supply shocks
Bryant (1980)	Instrument-money supply target	Demand for money-money supply, demand of reserves-supply of reserves	Money demand demand and supply of reserves shock
Tobin (1983)	Instrument-income target	Aggregate supply-aggregate demand (including price expectation)	Aggregate supply and price expectations shocks

included, leading the study into a complete aggregate demand-aggregate supply model.

From the discussion above we conclude that the theoretical framework must be specified according to the objective functions (e.g., stabilization of income or money supply), and to the types of simulations that will be performed (i.e., shocks).

For the present study, instruments have already been defined (cf. Chapter 1) as the tools that will be used to attain ultimate objectives. Therefore, income stabilization has been chosen as the objective since it is the macro variable that signals the overall stability performance of the economy, and it will usually be part of any macroeconomic policy package.[43]

The model on which our analysis will be based is described next. As will be evident below, the initial assumptions will be modified later in several ways so as to generate a set of different theoretical structures under which the analysis will be performed. This will enable us to capture the sensitivity of the results to different model specifications. This type of analysis could shed some light into the instrument choice problem within a context of uncertain structural parameters as well as uncertain models of economic behavior, which can be interpreted as shocks to the structure assumed(i.e.,

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43. I will follow Poole's methodology of keeping prices constant so that nominal and real income are equal.

economic model).

Table 2-5 describes the scope of our analysis, in the same fashion that previous studies on the subject were described in Table 2-4 above.

Table 2-5. Objective and Theoretical Framework in the Present Study

Key Relationship under study	Theoretical framework	Type of analysis
Monetary Income Instrument-target	Fix-price IS-LM stochastic (aggregated LM)	IS and LM shocks, under varying structural assumptions
	Fix-price IS-LM stochastic (disaggregated LM)	

II. DESCRIPTION OF THE THEORETICAL FRAMEWORK

Having in mind the objective of using monetary policy for income stabilization purposes, a framework of income determination comprising the monetary and the real sectors of the economy will be proposed. This will enable us to determine the monetary sector's role in the process of income determination and its sensitivity to outside factors, and to introduce the monetary policy procedures that are our main concern.

A. Description of Real and Monetary Sectors.

Starting from an equilibrium position, that is, a situation where both the monetary and the real sectors clear, the distortion caused by different disturbances or behavioral changes (that is, the comparative statics analysis), can be found . Therefore, the model will be specified for an equilibrium situation.

Using the simplest, closed-economy fixed-price IS-LM framework, the main relationships that compose and link the monetary and the real sectors will be depicted. Several assumptions will later be modified such that the analysis can be performed under different scenarios and capture the sensitivity of the results to several model specifications (i.e., different models of economic behavior).

Table 2-6. Real Sector Relationships

Basic Relationships:

$$C = f(Y) + U_c$$
$$I = g(r) + U_i$$
$$G = G_o$$
$$Y = C + I + G$$

Equilibrium Relationship (IS):

IS Curve: $Y = A + h(r)$

where:

C = consumption
Y = income
I = investment
G = government expenditure
r = interest rate
G_o = exogenous level of G
U_c, U_i = disturbance terms
$A = s[G_o + U_c + U_i]$
s = income multiplier of $G, U_c, U_i + \frac{1}{1-f'(Y)}$
$f'(Y) > 0,\ g'(r) < 0,\ h'(r) < 0$

NOTE: linear functions assumed.

We now proceed to define the real sector's elements and relationships: investment is an inverse function of interest rates; consumption is a direct function of income, and government expenditures are assumed exogenous. Hence, given that in equilibrium consumption (C), plus Investment (I) plus government expenditures (G) equal National Income (Y), the IS curve (reflecting the real sector's equilibrium) relates income and interest rates (r) inversely. It is also assumed that the behavioral relationships, investment and consumption are stochastic; that is, are affected by a disturbance term. (See Table 2-6). On the other hand, equilibrium in the monetary sector is such that money supply equals money demand. Although this sector will later be disaggregated and some assumptions will be changed, it is now described in a general form to place it in the context of income determination. The demand for money (i.e., demand for real balances) is assumed to be a direct function of income, (i.e., transaction balances) and an inverse function of interest rates (i.e., portfolio choice theory), and is also affected by a disturbance term. The money supply is assumed exogenous for the time being, an assumption that will be dropped when analyzing the monetary sector alone. Hence, we obtain the monetary sector's equilibrium as demand and supply are equated, yielding a positive relationship between income and interest rates, for a given level of money supply (i.e., an LM curve that contains the pairs of income and interest rates values that maintain the monetary sector in equilibrium). (See Table 2-7).

Table 2-7. Monetary Sector Relationships

$M_d = j(Y) + k(r) + U_{md}$	
$M_s = M_{s_o}$	Basic Relationships
$M_s = M_d$	
$Y = l(r) + B$	Resulting Equilibrium LM Relationship

where:

M_d = demand for money

M_s = money supply

Y = income

r = interest rate

M_{s_o} = exogenous value of money supply

U_{md} = disturbance term on money function

$B = n[M_s - U_{md}]$

$n = \frac{1}{j'(Y)}$

$j'(Y) > 0,\ k'(r) < 0,\ l'(r) > 0$

If we were to assume that the money supply is a (positive) function of interest rates, we would still have the positive relationship between income and rates depicted by the LM curve above, although the value of such relationship would be larger (i.e., $\frac{\Delta Y}{\Delta r}$ in the LM equation). The larger $\frac{\Delta Y}{\Delta r}$ arises because as interest rates rise, they lower the demand for money but also increase the supply. Hence, the discrepancy between money supply and money demand is greater than in the case where money supply does not respond to interest rates; therefore, for the monetary sector to clear , the new (greater) equilibrium income level needs to be greater than that necessary to clear the market when the only effect of interest rates is on the demand for money (See Figure 2-5).

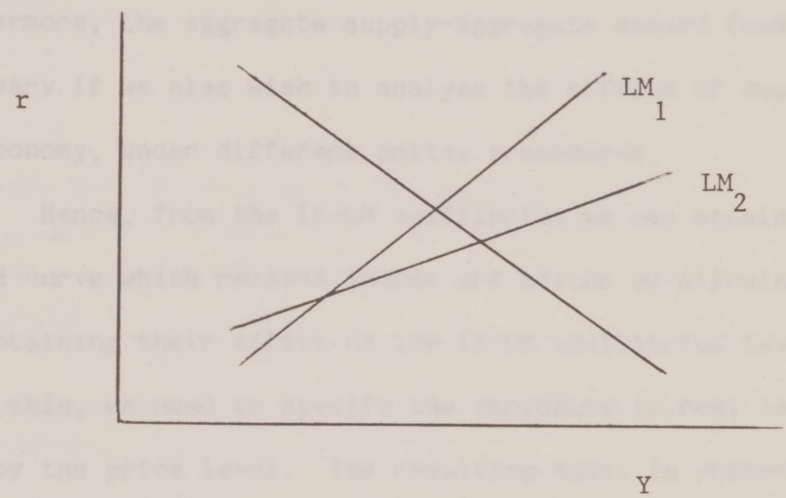
B. Aggregate Demand Relationship

Having determined the real sector's (IS) as well as the monetary sector's (LM) equilibrium loci, the demand side equilibrium for the economy will be determined by the level of income and interest rates that maintain both markets in equilibrium, that is, where the IS and LM intersect.

As long as we continue assuming fixed prices, it is enough to pursue the analysis within the above IS-LM context, evaluating the economic shocks according to their effect on the level of income that constitutes IS-LM equilibrium. However, as soon as we relax the fixed

Figure 2-5

The Monetary Sector Equilibrium
Relationship under Interest-Elastic and
Interest- Inelastic Money Supply.



Where; LM_1 = Money supply interest-inelastic assumed.

LM_2 = Money supply interest-elastic assumed.

price assumption, we need to work with the aggregate demand relationship, together with aggregate supply to obtain the resulting levels of income and prices under each of the cases analyzed. Furthermore, the aggregate supply-aggregate demand framework will be necessary if we also wish to analyze the effects of supply shocks on the economy, under different policy procedures.

Hence, from the IS-LM equilibrium we can obtain the aggregate demand curve which relates income and prices by allowing prices to vary and obtaining their effect on the IS-LM equilibrium level of income. To do this, we need to specify the variables in real terms dividing them by the price level. The resulting model is presented in Table 2-8.

As prices increase, the supply of real balances decreases and the demand for real balances thus exceeds supply. In order to obtain additional balances (on the demand side), the public will sell bonds (or any kind of interest-bearing assets) and cause interest rates to go up. Therefore, for any given level of income, equilibrium in the money market will require higher interest rates, such that the demand for real balances falls and equals supply (i.e., there has been a leftward shift in the LM curve). Figure 2-6 shows the LM shift due to the price increase, and the initial effect on the monetary sector as interest rates rise: from equilibrium A to point B on new LM. On the real sector's side, higher rates mean lower investment, so income will start

Table 2-8. Determination of Aggregate Demand

Real Sector	$C = f(Y) + U_c$ $I = g(Y) + U_i$ $Y = C + I\ G_o$
Monetary Sector	$Md = j(y)+k(r)+Umd$ $MS = \frac{MS_o}{P}$ $MS = Md$
	where: C, I, Y, G, Md, MS are in real terms MS_o = nominal supply of money P = price level

	Aggregate Demand
IS: $Y=h(r)+s(G_o+Uc+Ui)$	$Y = C + d(P)+U$
LM: $Y = l(r)+n(\frac{MS_o}{P}-U_{md})$	where $C = c(g,MS)$ $d(P)=F(l'(r),h'(r),j'(Y))$

to fall as investment is lower than savings at any interest rate above r_1; hence, production falls (implying a movement along the IS from point A to C). On the monetary sector, as income falls the demand for real balances does so, too, relaxing the pressure on interest rates and hence letting them fall (movement along new LM from point B to C). Such forces in both markets will continue until new equilibrium C is reached. Therefore, the new equilibrium is given along the same IS curve but intersecting with a new LM curve, to the left of the original one. The result is an inverse relationship between income and prices as shown in Diagram II of Figure 2-6. Then, the aggregate demand relationship will be of the form:

$$y = C + d(P) + U$$

Where $C=c(G,MS)$

$d(P)=F(l'(r),h'(r),j'(y))$

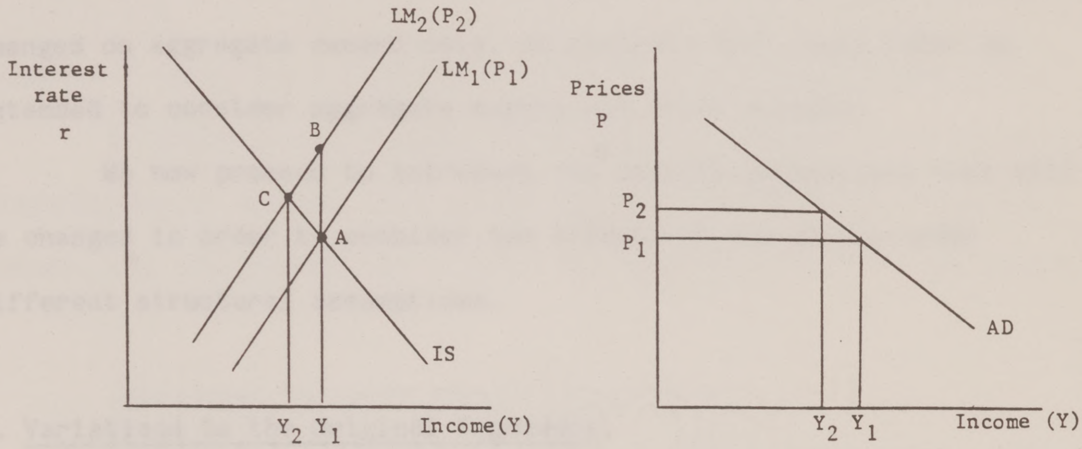
$U=u(Uc,Ui,Um)$.

where $l'(r)$ and $h'(r)$ are the LM and IS slopes (i.e., $\frac{\delta y}{\delta r}$ respectively).

From this aggregate demand equation it can be seen that changes in exogenous variables or disturbance terms from either sector will shift aggregate demand for any price level. Such a change will press prices up or down (depending on the case), the final effect on prices being determined by the shape and position of the aggregate supply curve. Furthermore, the final outcome -that is, whether income or

Figure 2-6

Derivation of the Aggregate
Demand Relationship

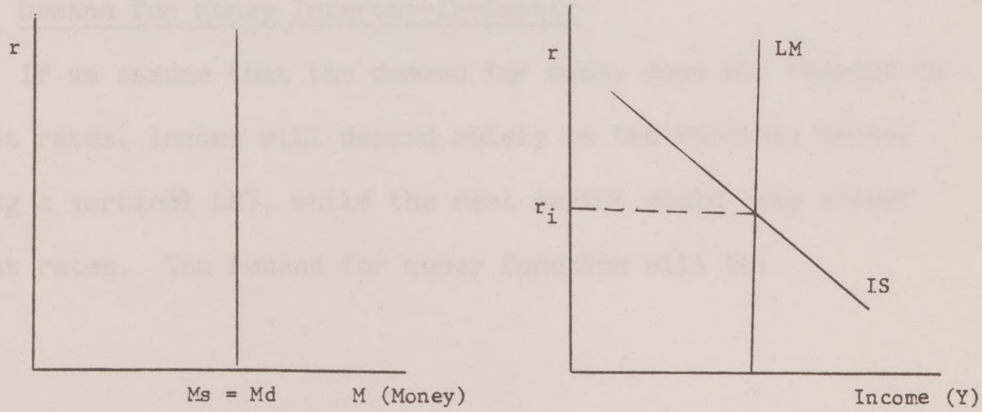


I. IS-LM Equilibrium.
Effect of a price increase.

II. Aggregate Demand
Relationship.

Figure 2-7

Income determination under the
assumption of an interest-inelastic
demand for money.



I. Monetary Sector

II. IS-LM Equilibrium

prices have changed due to an aggregate demand shift- will be determined by the shape of the aggregate supply curve (i.e., whether it is flatter or steeper) and its position (vis-à-vis full employment). However, our study will deal with the effects of shocks and unexpected changes on aggregate demand only, an analysis that could later be extended to consider aggregate supply and price changes.

We now proceed to introduce the various assumptions that will be changed in order to consider the effects of the shocks under different structural assumptions.

C. Variations to the Original Framework.

The assumptions and relationships that compose the initial framework of analysis will be modified to consider more complex situations, or situations proposed by different theories. Hence, we proceed to change some assumptions and derive the implications in terms of the IS-LM structure.

Demand for Money Interest-Inelastic

If we assume that the demand for money does not respond to interest rates, income will depend solely on the monetary sector (showing a vertical LM), while the real sector would only affect interest rates. The Demand for money function will be:

$$Md = j(y) + Um$$

and the LM equation becomes:

$$Y = B = n(\frac{Ms}{P} - Um)$$

Figure 2-7 shows the determination of income under this assumption.

In general, the steeper the LM is, the smaller the effects that disturbances or changes causing the IS curve to shift will have on income, and the greater the effects they will have on interest rates.

Investment Function Interest Inelastic

Another situation can arise when interest rates do not affect investment. In this case, the real sector alone determines the level of income. This is so because if the money supply changes, with the subsequent interest rate change, investment will not be affected and, therefore, the monetary sector's equilibrium will be forced back to the old level by greater movements in interest rates. In fact, interest rates would have to change sufficiently so as to cause the demand for money to equate the supply at the level of income determined by the real sector. The resulting investment equation will then be:

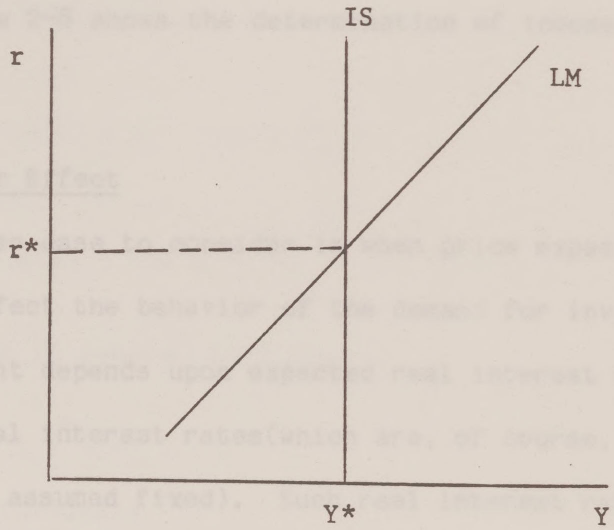
$$I = i_o + Ui$$

instead of:

$$I = g(r) + Ui$$

Figure 2-8

Determination of Income under an Interest-Inelastic Investment Demand Function



On the other hand, the IS equation will be:

$$Y = A$$

where $A = s\ (i_o + G + Uc + Ui)$

instead of:

$$Y = A + h(r)$$

where $A = s\ (G + Uc + Ui\ )$

Figure 2-8 shows the determination of income under this assumption.

Fisher Effect

Another case to consider is when price expectations have been formed and affect the behavior of the demand for investment; that is, when investment depends upon expected real interest rates, rather than on current real interest rates(which are, of course, equal to nominal if prices are assumed fixed). Such real interest rate variable will be defined as follows:[44]

44.

See Yiannis P. Venieris and F.D. Sebold, Macroeconomic Models and Policy. John Wiley and Sons, Inc., 1977, pp. 248-56.

$$\rho = \frac{(r-\dot{P}_e)}{(1+\dot{P}_e)}$$

where: ρ = real expected interest rate

r = nominal interest rate

P = actual price level

$\dot{P}_e$ = expected percent increase in price level.

Thus, the investment function we previously had:

$$I = g(r) + Ui$$

which equals:

$$I = g(\rho) + Ui \quad \text{under } \dot{P}_e=0,$$

becomes

$$I = g(\rho) + Ui = g\left(\frac{r - \dot{P}_e}{1 + \dot{P}_e}\right) + Ui$$

Similarly, the IS general form was:

$$Y = A + h(r)$$

where: $A=s(G+Uc+Ui)$.

Now, under the Fisher effect assumption it becomes:

$$Y = A + h_2 (r - \dot{P}_e)$$

where $h_2 = \frac{h}{1+\dot{P}_e}$

In other words, the IS will now be a function of r and price expectations. The new IS implies a shift and rotation from the old IS. The shift is reflected by the second term in the h_2 function, and the rotation is due to a different slope as reflected by the derivative of the h_2 function with respect to the interest rate (the new slope is equal to original slope divided by $(1+\dot{P}_e)$.

Figure 2-9 shows two IS schedules, one before and one after the Fisher Effect is introduced. Under the Fisher Effect assumption, the presence of price expectations lowers expected real interest rates and so investment increases, given the real return to capital. According to Fisher, investment demand will increase until the expected inflation rate has been introduced in the higher nominal rates. This is so because, as long as there is a difference between the nominal and the expected real interest rate, there will be a stimulus for investment demand to expand. On the monetary sector side, it is assumed that price expectations do not affect its basic relationships, since we have specified the demand for money as a function of its opportunity cost as measured by nominal interest rates.[45] Therefore, price expectations

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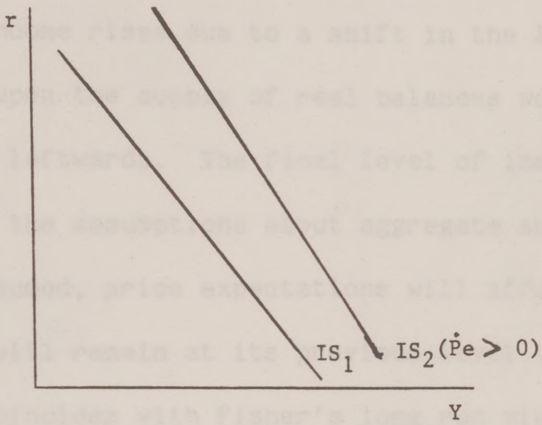
45. However, if we were to introduce a demand for money equation according to Milton Friedman's specification, where the prices of all the assets that are alternatives to holding money are included, then we would have price expectations affecting directly the demand for money function, as they measure the opportunity cost of holding money instead of physical goods.

Figure 2-9

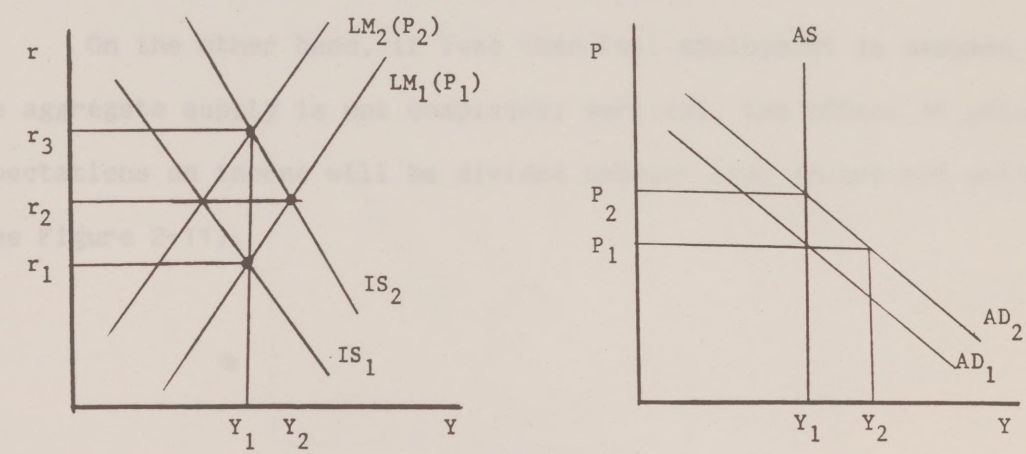
Determination of the IS curve
with Fisher Effect



IS_1 = without fisher effect $(\dot{P}_e=0)$
IS_2 = with fisher effect $(\dot{P}_e=0)$.

Figure 2-10

Fisher Effect: the Role of Expectations
in Investment and nominal Interest Rates,
assuming full employment.



I. IS-LM Equilibrium
IS_1 = initial IS
(no price expectations)
IS_2 = with positive
price expectations

II. Aggregate Demand (AD)
and supply (AS)

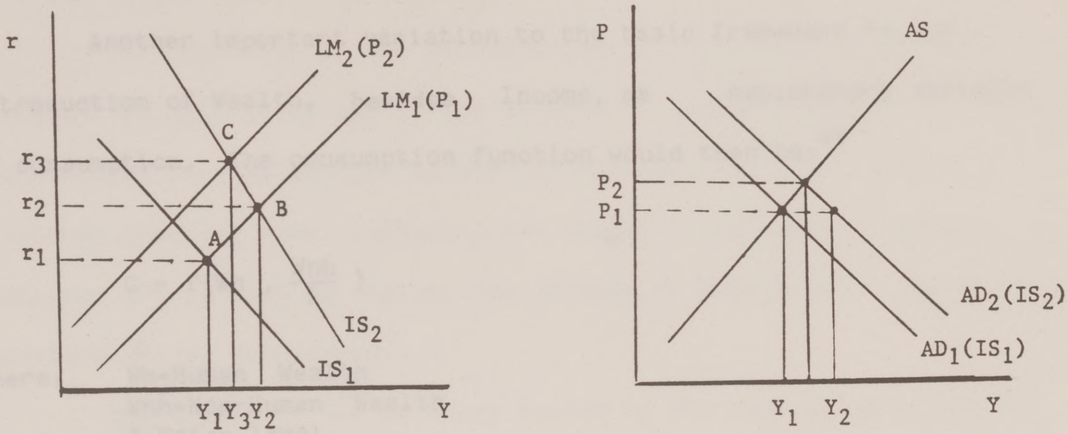
will not form part of the LM equation.

However, if we now allow prices to change, instead of having them fixed, as income rises due to a shift in the IS curve, prices may also rise, whereupon the supply of real balances would fall and the LM would thus shift leftwards. The final level of income and prices would thus depend upon the assumptions about aggregate supply. If full employment is assumed, price expectations will affect the price level, but real income will remain at its previous level (See Figure 2-10). This situation coincides with Fisher's long run view of the economy as expressed in the Quantity theory, where "MV=PQ" (M=money, V=velocity, P=prices and Q=output), and V and Q are assumed fixed in the long run. Therefore, a deviation of income from the long run level will result in prices rising until real income returns to its initial level. The net result in such case would be a new level of prices and nominal interest rates but the same level of output and real interest rates.

On the other hand, if less than full employment is assumed and the aggregate supply is not completely vertical, the effect of price expectations on income will be divided between real income and prices. (See Figure 2-11).

Figure 2-11

Fisher Effect: the role of Expectations in Investment and nominal interest rates, assuming less than full employment.

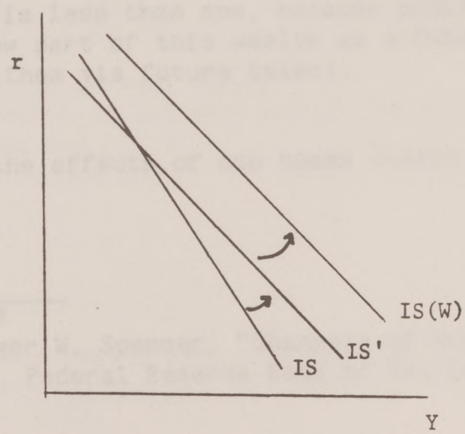


I. IS-LM Equilibrium

II. Aggregate Demand (AD) and Supply (AS)

Figure 2-12

The IS Curve with the Introduction of the Wealth Effect in Consumption



IS → no wealth effect in consumption

IS' → wealth effect in consumption

Wealth Effect on Consumption

Another important variation to the basic framework is the introduction of Wealth, besides Income, as explanatory variable of consumption. The consumption function would then be:[46]

$$C = f(Wh\ ,\ \frac{Wnh}{P}\ )$$

where: Wh=Human Wealth
Wnh=Non-Human Wealth
P=Price level.

Non-human wealth is defined as follows:

$$\frac{Wnh}{P} = \frac{MB}{P} + \beta\,\frac{GD}{P} + \frac{P_k K}{P}$$

where: MB=monetary base
GD=Government Debt (seen as part of private sector's wealth)
P_k =Price of real capital
K =stock of real capital
r =interest rate
β =proportion of the market value of government debt that is viewed as wealth by individuals. (β is less than one, because public may view part of this wealth as a future burden on them via future taxes).

Following Spencer, the effects of non human wealth on consumption will

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46. See Roger W. Spencer, "Channels of Monetary Influence: A Survey", in Review. Federal Reserve Bank of St. Louis, November 1974, pp. 8-26.

be classified as follows: the Real Balance Effect (through GD and $\beta \frac{G}{-}$) and the Equity Effect (through $P_k K$).

The inclusion of the wealth effect in the model may prove relevant to this study due to the effects of monetary policy on consumption through changes in the value of wealth. This will be explored when the model including the wealth effect on consumption (Chapter 3), is used to analyze the effects of shocks under diverse monetary policy procedures.

The effect of the monetary sector on the real balance portion of wealth arises directly from changes in MB, r or GD. On the other hand, the effects on the Equity portion of Wealth arise from changes in interest rates which in turn change the market value of real capital as reflected by stock prices. (i.e., stock price = $\frac{\text{face value}}{r}$).[47]

In terms of our IS-LM framework, the introduction of these effects into the consumption function implies that this function becomes:

$$C = f(Y , r , \frac{NW}{P}).$$

where: C=Consumption in real terms
NW = Net worth of private sector (herein measured by GD, MB and $P_k K$)
Y = Real Income and

47. For a discussion see: Douglas R. Pearce, "Stock Prices and the Economy", in Economic Review. Federal Reserve Bank of Kansas City, Nov. 1983, pp. 7-22.

$$\frac{\partial C}{\partial Y}>0, \frac{\partial C}{\partial r}<0, \text{ and } \frac{\partial C}{\partial NW/P}>0.$$

The IS would be flatter since as r increases (falls), the reduction (increase) in income to maintain equilibrium in the real sector, would be greater than in the original framework, due to decreased (increased) consumption. This can be verified as we obtain the IS that would correspond to a model with the wealth effect included. The basic framework specified the IS equation as follows:

$$Y = A + h\ (r)$$

where: $A = s(G + Uc + Ui)$

and $h'r$ less than zero.

Now, introducing the wealth effect as expressed by the following consumption function:

$$C = f_1\ (y) + f_2\ (r) + f_3\ (\frac{NW}{P}) + Uc$$

where: $f_1>0$, $f_2<0$ and $f_3>0$.

The IS equation becomes:

$$Y = A_2 + h_2\ (r)$$

where: $A_2 = s\ (G + f_3\ (\frac{NW}{P}) + Uc + Ui\ )$

and: $|h'_{2r}| > |h'_r|$

while: $A_2 > A$

Therefore, the new IS is flatter than the old one, but it has also shifted upwards due to inclusion of net worth. Figure 2-12 shows how the IS is affected when the wealth effect is introduced. The change in the slope of the IS comes from the fact that:

$$|h'_{2r}| > |h'_r|$$

and this would make the IS to rotate to IS'. The shift from IS' to IS(w) is given by:

$$f_3\left(\frac{NW}{P}\right)$$

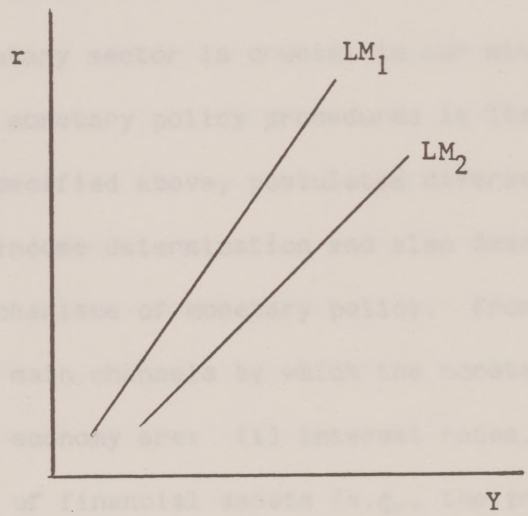
which made $A_2 > A$.

Interest Elastic Money Supply

This is the situation that arises if it is assumed that money supply depends on interest rate changes. This may result from a demand-determined element on the sources of reserves such as Borrowings, and/or from a possible Excess Reserves behavioral function, both of which may be depending on interest rates. If this were the case, the money supply will become a function of interest rates and, furthermore, it may also become a stochastic function. The effect of this assumption in terms of the IS-LM structure, is to make the LM flatter (i.e., enlarge its slope $\frac{(\Delta Y)}{(\Delta r)}$. (See Figure 2-13.)

Figure 2-13

The LM under an Interest-Elastic
Money Supply Assumption



LH_2 : includes interest-elasticity of the money supply.

When simulating different monetary procedures, the new LM slope may yield analytical results which are different than those under the original assumptions because, for the monetary sector to be in equilibrium, an interest rate change must be accompanied by a higher income change to bring the monetary sector back to equilibrium.

D. Disaggregation of the Monetary Sector

The monetary sector is crucial in our study's framework because the analysis of monetary policy procedures is its main concern . The framework, as specified above, postulates diverse assumptions on the demand side of income determination and also describes the basic transmission mechanisms of monetary policy. From such specifications we see that the main channels by which the monetary sector is linked to the rest of the economy are: (i) interest rates, as they reflect relative prices of financial assets (e.g., the price or opportunity cost of holding money), and affect investment and consumption; and (ii) money supply changes, since they reflect the relative availability of real balances with respect to the desirability to hold them, and affect wealth and price expectations(if price expectations depend upon money supply changes),

However, in order to analyze the implications of alternative monetary policy procedures (each procedure uses a specific instrument to attain target), a more disaggregated description of the monetary sector is required, particularly of the elements that determine the money supply. As the monetary sector is disaggregated, it will be possible to formulate behavioral relationships that more accurately describe the elements that determine the money stock. This will lead to a relaxation of the assumption of money supply exogeneity .

In fact, the money supply is not considered either exogenous or controllable by the authorities any more; rather, it is viewed as a variable which is endogenously determined as it depends on interest rates as well as on other constraints, such as institutional constraints and random behavioral changes within the financial system. To clarify this assertion, we need to proceed to the disaggregation of the money supply concept, starting with the sources of its creation.

First of all, we must place our analysis within the institutional framework in which money is created; for this purpose, the U.S. monetary framework will be used.[48]

D.1 Institutional Framework

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48. The framework that makes up for the determination of money supply may vary from country to country, and therefore the variables and relationships involved may be different from the ones we will use in this study. However, the present framework can always be changed to a specific case study.

The interaction of the monetary authority (The Federal Reserve), the Treasury, the banks and the public, will determine the amount of circulating money.[49] It will be assumed that there is only one type of financial intermediary (banks), which issues one type of deposits, (demand deposits), and which is required to hold a fraction of its liabilities at the Federal Reserve in the form of deposits (required reserves).[50] If total reserves held by banks exceed the amount of required reserves, computed as a percentage of the bank's deposit liabilities (i.e., the required reserve ratio determined by the FED), the difference is called excess reserves. Based on the total amount of reserves that banks hold at the FED, they can create funds and acquire income earning assets(i.e., securities and loans to the public). Therefore, credit creation by banks is based on an expansion of deposit liabilities, part of which must be maintained as reserves (required reserves), although the rest (excess reserves) can be lent out as credit. An increase in the amount of reserves in excess of what is required to back up deposits implies an increase in bank's capacity to increase credit. This assigns reserves an important role in the

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49. Money = Currency plus Demand Deposits.

50. Actually, in the U.S. banks can satisfy their reserve requirements with vault cash or by crediting their Federal Reserve Account. When we refer to Bank's reserves, we will assume that both are possible. Hence Total Reserves will include Reserve deposits at Fed plus vault cash.

determination of circulating money, while the power to set reserve requirement ratios gives the Fed an important leverage over deposit and credit creation.[51]

The Federal Reserve Banks (i.e., the monetary authority), on the other hand, issue deposit liabilities which are owned by commercial banks, by the Treasury and by foreign central banks or official institutions. They also purchase income-earning assets (U.S. government securities and loans to banks), issue Federal Reserve Notes and hold Gold Certificates and Special Drawing Rights.

Bank reserves are provided to the system mainly through the Fed's purchases of assets, either government securities (open market operations) or bank loans (discount operations). They are also provided by the Treasury's acquisition of gold from U.S. residents, which is monetized through the Fed by the issuing of Gold Certificates or by the Treasury's issuing of Treasury currency.

Total reserves plus currency in circulation form the monetary base or "high powered money" concept. The monetary base is also defined as the "net monetary liabilities of the Government (U.S. Treasury and Federal Reserve System) held by the public (commercial banks and nonbank public)."[52] Its importance in monetary policy analysis comes from the fact that it is composed of the primary sources

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51. See, "The Instruments of Monetary Policy," in Economic Review. Federal Reserve Bank of Kansas City, May 1984, p. 5.

of money creation; it can therefore be decomposed to show the items that provide or allow for additions to the circulating money supply. The figure for the monetary base is obtained from the Consolidated Balance Sheet of the Fed and the Treasury monetary accounts; it gives the total reserves plus currency in circulation, as well as the items that add up to them. Table 2-9 shows the elements of a Consolidated account, and distinguishes the factors that provide (i.e., are sources of) reserves, as well as the factors that absorb (i.e., use) reserves from the monetary system.[53]

Using the Table's notation, the monetary base is defined according to its net sources as:

$$\underline{MB = Sma + BOR + Flt + OA + GS + TCO + SDR - TCH - Dep - NW}$$

where the factors that are being added are those supplying reserves (i.e., adding to the monetary base and hence to money supply), while the ones being substracted are those using reserves.

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52. Jerry L. Jordan, "Elements of Money Stock Determination". in Current Issues in Monetary Theory and Policy. Thomas Havrilesky and John Boorman, 1976, p. 264.

53. See Ralph Bryant, Controlling Money, The Federal Reserve and Its Critics. Brookings Institution, Washington, D.C., 1983.

Table 2-9. U.S. Monetary Authorities Balance Sheet: Sources and Uses of Reserves. (Consolidated balance sheet of the Fed and the Treasury).

SOURCES OF RESERVES	USES OF RESERVES
Federal Reserve Credit:	Currency in Circulation (CUR)
U.S. Govnt. Securities (Sma)	(outside Fed and Treasury)
Govnt.Securities	
Agencies' Securities	Treasury Cash Holdings (TCH)
	(Fed notes held by Treasury plus free gold)
Acceptances (A)	Deposit liabilities of Fed(Dep)
	a)Due to Treasury (DEG)
	b)Due to Foreigners (DEF)
Loans to Banks (BOR)	c)Due to Others (DEO)
(Discounts and Advances)	
Float (Flt)	Other Accounts (NW)
Other Assets(OA)	Fed's Liabilities and capital
Gold Stock (GS)	
(Entire treasury gold stock)	
Treasury Currency Outstanding (TCO) (monet. liabilities)	
	Member Bank Reserves(RT)
	(Reserves plus vault cash)
SDR certificate accts (SDR)	

Notes: Free gold=total gold stock minus gold certificates held by the Fed=part of the gold stock that does not have corresponding gold certificates. TCO=monetary liabilities of the treasury, that is, stock of currency issued by the treasury and held by the public, Fed or depositary institutions.

A brief explanation of each of the items that compose the monetary base is useful.

Factors Supplying Reserves.

Government Securities (Treasury) and Agency Securities (Federal government agencies) constitute the major source of reserves in the american financial system. Through open market operations(OMO), securities are sold and bought to and from the banking system and so absorbing or adding to reserves that will increase or decrease the monetary base and hence the money supply.

If government securities are sold to banks, Sma decreases and so does RT (i.e., the Bank's reserve account is debited by the value of the securities sold). If the Fed decides to buy securities from the banks, Sma increases and so does RT. Open market operations are then reflected in Sma, and the Fed has a close control over Sma by performing OMO in the desired direction.

However, a different situation might arise if Sma is increased due to new debt created to finance government expenditures. In this case, Sma will increase and so will DEG, the Treasury deposits at the Fed. The effect of this new debt on money supply will be complete when the Treasury spends the new funds, transferring reserves from DEG to RT. Furthermore, if the Fed decides to sell the newly created securities to the commercial banks, this would decrease the banks' reserves at least while the Treasury has not spent the deposits matching the new debt. After the Treasury has used such deposits, reserves will be back to the initial level and so will the monetary base. However, interest rates will have probably risen.

Acceptances (A) are documents arising from a time draft due to international trade, and they can be rediscounted at a commercial bank when accepted.

Member Bank Loans (BOR) are the result of borrowing either in the form of discounts (a bank rediscounts a paper it owns) or advances (loans properly), all of which are subject to the discount rate. Advances are the most common of these types of loans.

The float (Flt) arises when the Fed makes use of his own deposits at member banks. It constitutes the increased reserves available to the banking system from the time a Fed's check against its own account in a member bank is in the power of another member bank, to the time it is collected. In other words, the float constitutes a financial magnitude generated by timing discrepancies in the payment mechanism. The Fed tends, on average, to credit the reserve accounts of receiving banks sooner than it debits the reserve accounts of paying banks. Hence, the effect is to increase reserves.

The gold stock (GS) is in the possession of the U.S. Treasury and not owned by the Fed. When the Treasury buys gold from a resident, GS increases and bank's reserves (RT) increase too. Then, The Treasury issues Gold Certificates which are claims against such gold stock and these are held at the Fed. As this is done, the gold stock is being monetized since gold certificates are deposited at the Fed at the same time Treasury Deposits at the Fed are credited. However, no further change in total reserves occurs. (Note the appearance of Gold Certificates on the right hand side of Table 2-9). There may be part of the gold stock against which no certificates have been issued (i.e., have not been monetized), a portion known as free gold which is included under "Treasury Cash Holdings"(TCH).

The Treasury Currency Outstanding (TCO) consists of all current monetary liabilities of the U.S. Treasury (coins and notes), whether held by the public, depositary institutions or the Federal Reserve. If the Treasury increases its outstanding currency, since vault cash can be held as reserves by depositary institutions, there would be an increase in total reserves. When new currency is created by the Treasury, TCO and DEG are increased, but as it is spent the change in DEG is transferred to RT.

The Special Drawing Rights (SDR) are instruments allocated by the International Monetary Fund to its members, and are part of the country's international reserves. They can be used to exchange foreign currencies among countries.

Factors Absorbing Reserves.

Treasury Cash Holdings (TCH) includes currency holdings by the Treasury plus Free Gold (i.e., Gold Stock minus Gold Certificates), the first of which absorbs reserves by holding currency that otherwise would be in public hands; on the other hand, a change in free gold will always be accompanied by an offsetting change in DEG, the net effect in total reserves thus being zero.

Therefore, the Federal Reserve's liabilities in form of Deposits are due to:

i) The Treasury (DEG)

ii) Foreign Central Banks or Treasuries (DEF), and

iii) Others (DEO)

In the first case, as the Treasury pays out from its deposits at the Fed (i.e., as DEG falls) ceteris paribus, the Bank's reserves will rise and the money supply too. Treasury deposits (DEG) will also be affected by tax collection (although in this case the net effect in RT is zero, as taxes are spent by government); by treasury borrowing from non bank public (the net effect in RT being zero too); and by Treasury borrowing from commercial banks (the net effect in RT being zero too, but money supply increasing if excess reserves are not zero when borrowing occurs).[54] Foreign deposits, on the other hand, arise from international transactions and capital flows. The relevance and impact of these in Total Reserves will vary between countries.

The U.S. Monetary Authorities Consolidated Balance Sheet for July 1983 is shown in Table 2-10.

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54. For a simple explanation of the accounts' changes, see Ritter and Silber, <u>Principles of Money and Banking.</u> 4th edition, Basic Books, 1974.

Table 2-10. U.S. Monetary Authorities Consolidated Balance Sheet. July 1983 . (End of month figures)
Millions of Dollars

SOURCES OF RESERVES		USES OF RESERVES	
Fed Credit		Currency (CUR)	159953
US Gov Secur (Sma)			
Gov Securities	144255		
Agencies Sec.	8880		
Acceptances	0	Treas. Cash(TCH)	515
Loans(BOR)	1113	Deposits with	
Float(Flt)	1066	Fed Banks of	
Other Assets(OA)	8597	Treas (DEG)	3815
Gold Stock(GS)	1113	Foreign(DEF)	369
SDRs (SDR)	4618	Other(DEO)	566
TreasCurrncyOut(TCO)	13786	Required clearing	
		balances	830
		Reserve Accounts	
		Member Banks (RT)	22201
Total Sources	163893	Total Uses	163893

Source: Federal Reserve Bulletin. Board of Governors of the Federal Reserve System. Washington, D.C. August 1983.

It depicts the elements that compose the monetary base, and gives a good picture of those elements that are more important in the total amount of Bank's Reserves.

Characteristics of the Determinants of the Monetary Base

The elements that determine the Monetary base may or may not be controllable by the Fed. Table 2-11 characterizes them according to the degree of the Fed's control over them.

Table 2-11. Factors that determine the Monetary Base according to the Degree of Fed's Controllability.

CONTROLLED	SEMI-CONTROLLED	UNCONTROLLED
US Govnt Securities (Sma)	Borrowings (BOR)	Float(Flt)
		Treas Cur Outs(TCO)
		Treas Cash (TCH)
		Dep at Fed by Treas (DEG)
		SDRs (SDR)
		Gold Stock (GS)
		Foreign Depos(DEF)

Controllability in this table is determined only with reference to the potential to control such element based on its own definition. No other criteria was used.

Although most factors determining the monetary base are not under close control by authorities in the U.S. government securities (Sma) give the Fed enough power to offset movements in the other elements, since represent the largest component of the sources of reserves, and are closely controllable.

However, the existence of Borrowings (BOR), which allow commercial banks to obtain funds besides those arising from open market operations, and the effect of interest rates on the demand for such borrowings can have important implications for the Fed's capacity to control the overall amount of available reserves, although the Fed could exert some control over those borrowings by changing the rate it charges them (i.e., the discount rate).

It has been argued and empirically tested that borrowings respond to the differential between the Federal Funds rate and the discount rate.[55]

The response of borrowings to changes in the federal funds rate will make the monetary base an endogenously determined variable, but their response to the rates' differential would signal some possibility of the Fed's control over them. Santomero quotes Peter Keir's[56] conclusion about the relationship between Borrowings and the interest rates.

> "The results strongly suggest that the level of borrowing is significantly correlated with the spread between the two rates, but not with the absolute level of those rates."

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55. See Robert H. Rasche, "A Review of Empirical Studies of the Money Supply Mechanism", in Review. Federal Reserve Bank of St. Louis, July 1972, pp. 11-19; and Anthony M. Santomero, "Controlling Monetary Aggregates: The Discount Window," in Journal of Finance, Vol. 38., No. 3, June 1983, p. 827.

56. Peter Keir, "Impact of Discount Policy Procedures on the Effectiveness of Reserve Targetting," in Santomero A., op.cit., p. 829.

He also notes that Kier's results show greater response of Borrowings to the interest rate differential after 1979, that is, after the operating procedures were changed to focus on reserve growth rather than on interest rates. This is a result of such a shift in policy procedures since after 1979 interest rates have been volatile, while this was not the case before that year due to the nature of monetary policy. Therefore, it is expected that evidence on the relationship between the monetary base and interest rates will be clearer after 1979 than before (i.e., if interest rate movements were very limited before 1979, it would be difficult to observe a conclusive relationship).

On the other hand, among the elements that determine the banks' reserves, some depend upon international transactions. Table 2-12 further classifies them according to the nature of their determining factor (i.e., domestic or foreign).
This distinction may prove relevant in certain countries depending on the role they play in the international arena, on the level of development of the Government Securities' market, on the type of foreign exchange system, and on the magnitude of their international transactions relative to domestic expenditure.

To summarize, the determination of the amount of reserves available to commercial banks can be expressed as the identity shown in Table 2-13.

Table 2-12. Components of the Monetary Base by Origin (Domestic vs. Foreign) and Degree of Control.

Control / Origin	Controlled	Semi-Controlled	Uncontrolled
Domestic	SMA	BOR	FLOAT TCO, TCH, DEG
International			SDR's DEF

Table 2-13. Reserves Identity.

TOTAL RESERVES=FACTORS SUPPLYING-FACTORS ABSORBING RESERVES RESERVES

TOTAL RESERVES	=	Securities Borrowings Float Gold stock SDRs Treasury currency outstanding	- Currency Treasury Cash Government Dep Foreign Deposits Other Deposits

The relevance and importance of each economic agent determining reserves (e.g authorities, banks, foreign, etc) will vary from one case to another depending upon economic, political, and social factors, as well as on the stage of development of the country under analysis. In well as on the stage of development of the country under analysis. In the U.S. case, monetary authorities play a predominant role by using government securities as the main "controlled" mechanism to increase or decrease monetary reserves. However, it can be said that, in general, a behavioral or institutional change that may lead to a change in the "uncontrollable" sources of reserves will be transmitted as a shock on the supply side of the monetary sector. The effects of such a shock will depend upon the type of monetary procedures being used. It is also important to note that, even in the U.S. case, there are other elements that may affect the money supply, such as the type of exchange rate system and the relations with other countries. U.S. monetary policy can affect the dollar exchange rate under the flexible exchange rate system (as it presently does), which in turn feedbacks into the system through the real sector and hence into the level of income and interest rates. This will in turn affect monetary and real variables, depending upon the type of monetary policy procedure being used.

D.2 Model Specification: The Monetary Base, Money Supply, and Reserves

Finally, in order to introduce the disaggregated monetary sector into the IS-LM model, it is necessary to link the monetary base to the money supply, a task which we now undertake.

The monetary base has been defined as:

$$(1) \quad MB = TR + CUR$$

Where: TR = Total Reserves

CUR= Currency in Circulation.

Total Reserves, on the demand side, are divided into required reserves and excess reserves. Therefore:

$$(2) \quad TR = RR + ER$$

Where: RR = Required Reserves

ER = Excess Reserves

It is known that required reserves are a portion of demand deposits, such that:

$$(3) \quad RR = rrD$$

where: rr = required reserve ratio

D = Demand deposits

On the other hand money supply is defined as follows:

$$(4) \quad M = CUR + D$$

For the moment, it is assumed that currency (CUR) is a fixed portion (c) of demand deposits (D), such that:

$$(5) \quad CUR = c\,D$$

From those relationships, Money Supply (M) and Total Reserves (TR) will be linked using the relationship between Demand Deposits (D) and required reserves(RR). This is done as follows:

Equations 2, 3 and 5 are introduced into equation 1:

$$(1a) \quad MB = rrD + ER + cD$$

Solving for D yields:

$$(6) \quad D = \frac{(MB-ER)}{(c+rr)}$$

Equations 5 and 6 are then substituted in 4:

$$(4a) \quad M = \frac{(MB-ER)\ (1+c)}{(c+rr)}$$

The supply side of the monetary base is defined in 7:

$$(7) \quad MB = Sma+BOR+Flt+TCO+GS+SDR$$
$$-TCH-DEG-DEF-DEO$$

Hence, incorporating 8 into 4a, the relationship between MB and M is obtained giving the following, very simplified, expression:[57]

$$(4b) \quad M = (Sma+BOR+.....-ER)\frac{(1+c)}{(c+rr)}$$

---

57. Very simplified, because of the many implicit assumptions under which such formula was arrived at (e.g., a single type of rr, exogenous CUR, BOR, etc.).

The above model specification could be extended to include different types of deposits, required reserve ratios, interest rate effects on different items that compose M, and even stochastic disturbance effects on M via some of its elements.[58] We will introduce some of these variations later (see "Model Specification:New Assumptions", below).

D.3 Model Specification: LM determination

The money supply formula, in terms of its primary sources, together with the demand for money equation as specified in the IS-LM model above, will be taken together in order to determine the equilibrium of the monetary sector.

The Monetary Sector

Demand for money: $M = j(Y) + k(r) + Umd$

Money Supply: $M = \frac{(1+c)}{(c+rr)}\,(Sma+BOR+..)$

Equilibrium: $Md = Ms$

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58. For a very detailed description of possibilities, see Havrilesky and Boorman. Money Supply, Money Demand and Macroeconomic Models, Second Edition, Harlan Davidson Inc., 1982.

This formulation of the monetary sector relationships continues to assume that the money supply is an exogenous variable, since the different elements that determine it are so assumed.

From the monetary equilibrium above, the LM is derived and results in the following equation:

$$Y = \frac{m_o(MB-ER)}{j'(y)} - \frac{k(r)}{j'(y)}$$

where $m_o = \frac{(1+c)}{(c+rr)}$,

$MB-ER = Sma+BOR+.....-ER$

$j'(y)$ = derivative of $j(y)$ with respect to y;

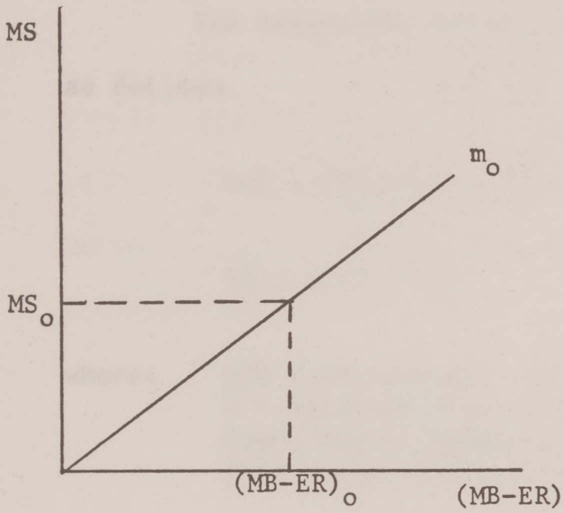
assuming $j(y)$ is a linear function of y.

The diagrammatic representation of this equilibrium, and hence the derivation of the LM appears in Figure 2-14.

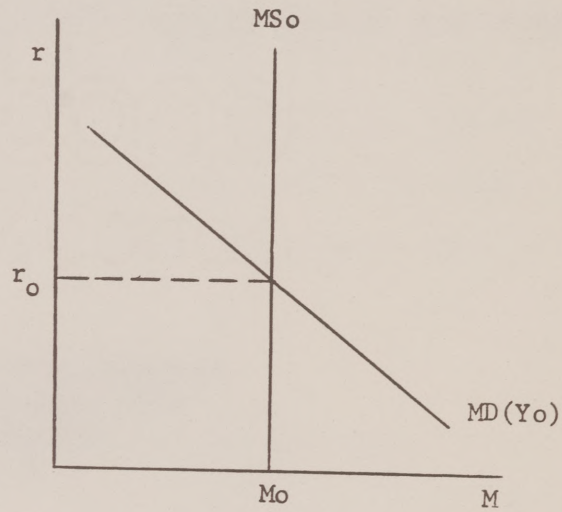
Part A of Figure 2-14 shows the determination of the Money supply. Part B shows the equilibrium of demand and supply; the demand for money schedule is drawn for a given level of income and assumes the disturbance Umd is zero. The money supply is fixed at the level determined by Part A.

Figure 2-14

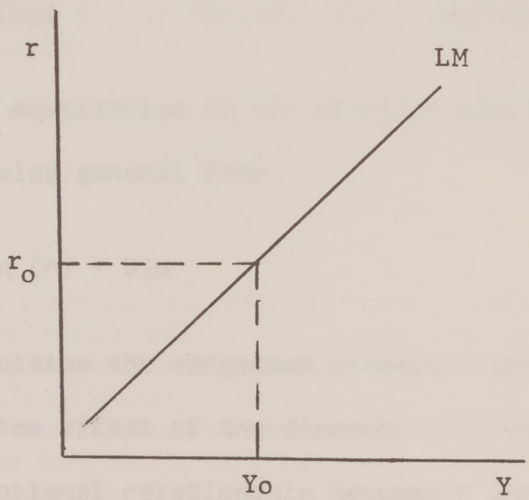
Monetary Sector Equilibrium
with a disaggregated monetary sector.



A. Determination of Money Supply (MS), given the monetary base (MB) and excess reserves (ER).



B. Money Supply (MS) and Demand (MD)



C. Monetary Sector Equilibrium.

D.4 Model Specification: New Assumptions

The next step is to vary some of the assumptions implicit in the monetary sector.

Determinants of Borrowings (BOR) and Excess Reserves(ER).

The behavioral relationships determining BOR and ER will change as follows:

BOR = f(r-rd) + Ubor

ER = e(r) + Uer

where: BOR = Borrowings ER = Excess Reserves
r = interest rate rd= discount rate
Ubor, Uer = random disturbances
f'(r - rd)> 0 and e'(r) < 0

By adding these assumptions, the money supply becomes a function of the interest rate, the resulting equation being:

MS = mo [Sma + ...+ f(r-rd)- e(r) + Uer+Ubor]

Hence, the equilibrium of the monetary sector will be given by an LM of the following general form:

$$Y = \Omega + \eta\ (r) + Ulm$$

Hence Ω contains the exogenous elements that determine the monetary base and the effect of the discount rate on the money supply, while η is the functional relationship between r and Y that would maintain the monetary sector in equilibrium. The latter is being determined by the relationship between interest rates and money supply through BOR and ER, as well as by the relationship between interest rates and money demand. The Ulm term includes the random disturbances Uer, Ubor and Umd.[59]

Note that $\eta'(r)$ is the slope ($\frac{dy}{dr}$) of the LM curve, and it is greater than in the previous (original) case. The monetary sector's equilibrium can now be viewed in Figure 2-15, where the LM is shown before and after the interest-elasticity of the money supply was incorporated.

Currency and Demand Deposits as separate functions

The demand for money specification will be substituted by two different functions, one for currency (CUR) and one for deposits (DD). This allows currency and deposits to be affected differently by Y, r and random disturbances; furthermore, we allow for the possibility of shifts from DD to CUR or viceversa (e.g., a shift from DD to CUR will

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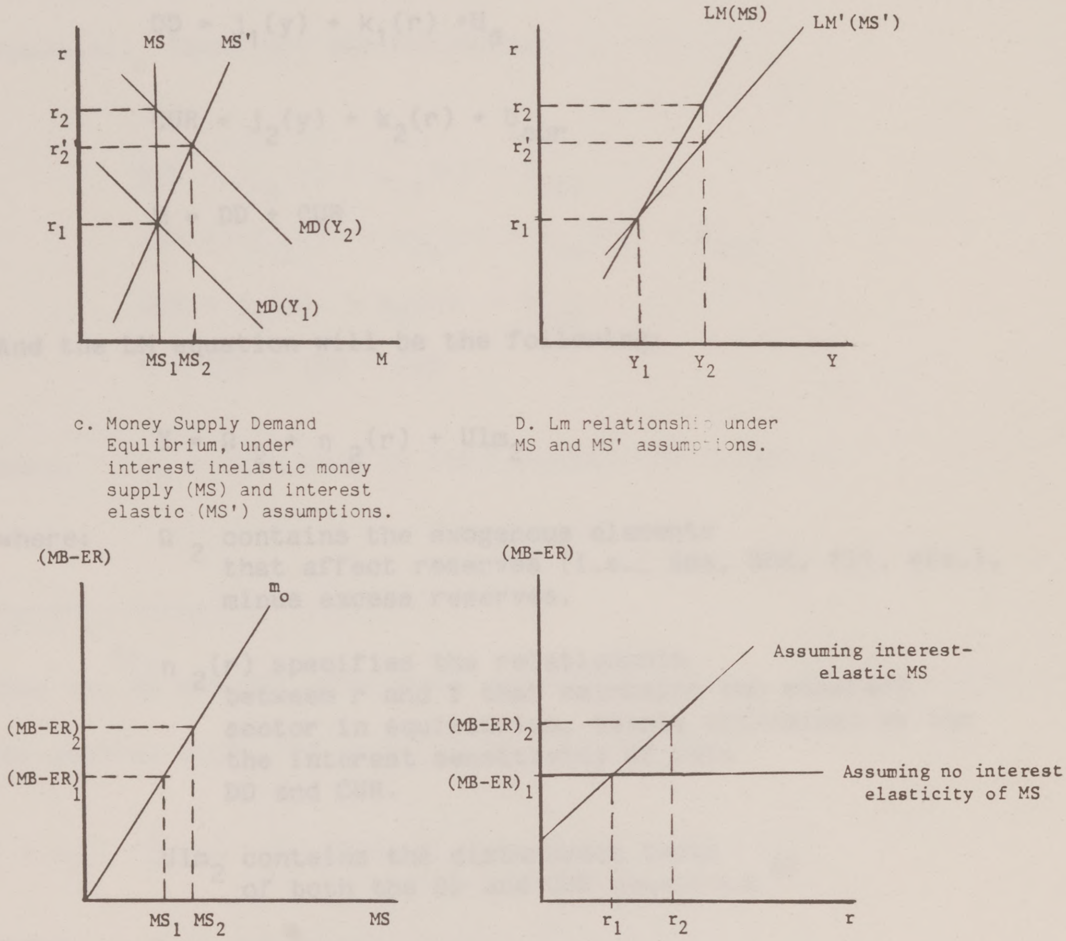
59. The LM formulation, showing each of the new assumed relationships is the following:
$Y = m(m_o(C)+m_o f(r-r_d)-m_o e(r)-k(r)-Umd+Uer+Ubor)$
where: m_o= reserve multiplier.
C=monetary base sources excluding borrowings.
m= reciprocal of the derivative of j(y) with respect to Y (assuming j(y) is linear).

Figure 2-15

Determination of Money Supply and
the Monetary Sector Equilibrium (LM) under
an inelastic Money Supply (MS) and
and elastic one (MS').



c. Money Supply Demand Equlibrium, under interest inelastic money supply (MS) and interest elastic (MS') assumptions.

D. Lm relationship under MS and MS' assumptions.

B. Multiplier Relationship Money and (MB-ER)

A. Relationship between (MB-ER) and Interest Rates.

be represented by a positive disturbance in the currency function, which is equal to a negative disturbance in the deposit function). This may make a difference when we get to analyze diverse monetary policy procedures in Chapter 4. The general specification of the new assumptions is the following:

$$DD = j_1(y) + k_1(r) + U_d$$

$$CUR = j_2(y) + k_2(r) + U_{cur}$$

$$M = DD + CUR$$

And the LM equation will be the following:

$$Y = \Omega_2 + \eta_2(r) + Ulm_2$$

where: Ω_2 contains the exogenous elements that affect reserves (i.e., Sma, BOR, Flt, etc.), minus excess reserves.

$\eta_2(r)$ specifies the relationship between r and Y that maintains the monetary sector in equilibrium, mainly determined by the the interest sensitivity of both DD and CUR.

Ulm_2 contains the disturbance terms of both the DD and CUR equations.[60]

Interest- Bearing Demand Deposits.

A new type of deposits will now be introduced. They are interest-bearing demand deposits (IDD), which bear the interest rate r_i. In this case the model will contain a different equation for each: the demand for IDD and the demand for NDD (not-interest bearing demand deposits). These are the following:

$$NDD = j_{1a}(y) + k_{1a}(r) + U_{ndd}$$

$$IDD = j_{1b}(y) + k_{1b}(r) + k_{1c}(r_i) + U_{idd}$$

$$CUR = j_2(y) + k_2(r) + U_{cur}$$

$$M = CUR + IDD + NDD$$

where: r_i = rate paid on IDD, assumed lower than r.

60.
When the DD and CUR equations, as specified above, are introduced into the general form, the detailed LM form becomes:

$$Y = m_2 \left[\frac{C_2}{rr} \right] - m_2 \; (rrk_1(r) + k_2(r)) - m_2(rrUd+Ucur)$$

where: m_2 = reciprocal of the derivative of

$rrj_1(y) + j_2(y)$, (assuming linear model), and

C_2 = MB - ER.

It is now appropriate to make certain qualifications to these demand for money equations. Initially, it will be assumed that the rates paid on IDD are controlled (i.e., have ceilings); therefore, the opportunity cost of holding this type of demand deposits is $(r-r_i)$, which enables us to specify the demand for IDD as follows:

$$IDD = j_{1b}(y) + k_{1b}(r-\bar{r}_i) + U_{idd}$$

where: $\bar{r}_i$ is the r_i ceiling value(a constant).

Hence , the demand for deposits becomes:

$$DD = N\ DD + I\ DD$$

$$DD = j_{1a}(y) + j_{1b}(y) + k_{1a}(r) + k_{1b}(r-r_i) + U_{ndd} + U_{idd}$$

or, more generally,

$$DD = J_1(y) + K_1\ (r) - k_{1b}(\bar{r}_i) + V_{dd}$$

Also the LM equation will be:

$$Y=\Omega_3+\eta_3(r)+Ulm_3$$

where: Ω_3 contains the exogenous elements that affect reserves, including $k_{1b}(\bar{r}_i)$.

η_3 specifies the relation between r and y that maintains monetary equilibrium which is mainly determined by the interest sensitivities of IDD, NDD and CUR.

Ulm_3 contains the disturbance terms of NDD, IDD and CUR.[61]

We expect that $|K_1'(r)| < |k_1'(r)|$ [62] since as r changes, the opportunity cost of DD increases but part of such cost can now be avoided by shifting from NDD to IDD. However, should IDD not exist, there is no way to avoid such cost or any part thereof, and hence people may respond stronger (decreasing DD more) to interest rate hikes. In other words, with the possibility that some demand deposits pay interest, the interest elasticity of total demand deposits has been lowered. On the other hand, as r increases there may be a shift from CUR to IDD, since IDD have liquidity properties close to those of CUR and by shifting to IDD part of the opportunity cost of CUR is avoided.

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61. The specific LM equation when introducing IDD, NDD and CUR equations is as follows:

$$Y=[m_3C_3] + m_3[rrK_1(r)+k_2(r)]+m_3[rrV_{dd}+U_{cur}]$$

where: m_3=reciprocal of derivative of $[rrJ_1(y)+j_2(y)]$, assuming linear functions. And

$$C_3=MB-ER.$$

$$m_3[rrK_1(r)-k_2(r)]=\eta_3$$

$$m_3[rrV_{dd}-Ucur]=Ulm_3$$

62. Where the former is the interest sensitivity(derivative with respect to the interest rate) of the new demand deposit function, while the latter is the interest sensitivity of the demand deposit function when deposits do not bear interest.

This contributes to the lower interest-elasticity of DD under the present model, and also means that the interest elasticity of CUR under the present assumptions (i.e., that IDD exist) is greater than under the previous case (i.e., when IDD did not exist).

Now, if no ceilings on rates were assumed (i.e., deregulation of interest rates), the opportunity cost of IDD would tend to zero, that of NDD would be r and the interest elasticity of NDD would probably be greater than before, as will that of CUR. As the interest rate increases, some NDD and CUR will become IDD, therefore, the interest elasticity of DD (NDD+IDD) will tend to be even lower than with ceilings (maybe tend to zero). This would also imply a steeper LM curve, with all the appropriate implications. Table 2-14 summarizes the cases in which: a) there are no IDD's, b) there are IDD's with ceilings on r_i, and c) there are IDD's without regulations limiting r_i's.[63]

63. See Tobin, op.cit., 1983.

Table 2-14. Demand for Money Functions under Assumptions of Non-interest Bearing Demand Deposits (A), Regulated Interest-Bearing Demand Deposits (B), and Non-Regulated Interest-Bearing Demand Deposits (C).

Cases	Equations
A Non-interest bearing demand deposits	$DD = J_1(y) + K_1(r) + Ud$ $CUR + J_2(y) + K_2(r) + U_{cur}$
B Regulated interest-bearing demand deposits	$NDD + j_{1a}(y)+k_{1a}(r)+U_{ndd}$ $IDD = j_{1b}(y)+k_{1b}(r-\bar{r}_i)+U_{idd}$ $CUR = J_2(y) + K_2(r) + U_{cur}$
C Non-regulated interest-bearing demand deposits	$NDD = j_{1a}(y)+k_{1a}(r)+U_{ndd}$ $IDD = j_{1b}(y)+k_{1b}(r-r_i)+U_{idd}$ $CUR = J_2(y) + K_2(r) + U_{cur}$

Where: k'_{1a}, k'_{1b}, k'_2 are < 0

$$| k'_{1a}(r) | > |k'_{1b}(r) |$$

$$k'_{1b}(r-r_i) \rightarrow 0 \text{ in case C}$$

$$| K'_2(r) |_{\text{case A}} < | K'_2(r) |_{\text{cases B and C}}$$

$$| k'_{1a}(r) |_{\text{case C}} > | k'_{1a}(r) |_{\text{case B}}$$

Integration of cases: Borrowings and Excess Reserves interest elastic plus interest bearing demand deposits.

If the assumptions of an interest-elastic money supply and interest bearing demand deposits are introduced together in the model, the former partly offsets the Lm steeperness caused by the latter. The resulting LM equation would be the following:

$$(1) Y=m_3[C_4+f(r-rd)-E(r)-rrK_1(r)-k_2(r)+Ubor-Uer-rrU_{dd}-U_{cur}]$$

Hence: $Y=\Omega_4+m_3(f(r-rd)-e(r))-\eta_3(r)+Ulm_3+m_3(Ubor-Uer)$

Where: m_3=reciprocal of $[rrJ_1(y)+j_2(y)]$ with respect to Y, assuming linear functions.(same as in case C above).

$\Omega_4=m_3C_4$

C_4=Sma+Flt+TCO+GS+SDR-TCH-DEG-DEF-DEO

$\eta_3(r) = m_3[rrK_1(r)-k_2(r)]$

The following relationships would apply in our final LM equation (1):

$BOR = f(r-rd) + Ubor$
$ER = e(r) + Uer$
$NDD = j_{1a}(y) + k_{1a}(r) + U_{ndd}$
$IDD = j_{1b}(y) + k_{1b}(r) + k(r_i) + U_{idd}$
$CUR = j_2(y) + k_2(r) + U_{cur}$
$MB = RT + CUR = rrDD + ER + CUR$
MB = Sma + BOR + flt + TCO+GS+SDR-TCH-DEG-DEF-DEO

Comparing this LM equation with the previous one (under case c.), we see that here the components f(r-rd) and e(r) enlarge the slope of the

LM (i.e., $\frac{\Delta Y}{\Delta r}$) and hence make it flatter than in case c.

III. CONCLUSIONS

This chapter has specified the theoretical framework that will serve as the basis for the upcoming monetary policy analysis. Such framework has been modified by the introduction of different hypotheses or assumptions with respect to the relationships included in the initial specification. The result has been a set of alternative frameworks, each with certain general assumptions and one so particular that makes it different from the rest.

The framework on each alternative case has been summarized in the respective IS and LM relationships, which will be the key elements in the analysis of the effects of economic shocks on the level of income under alternative policy procedures.

The first set of frameworks, in which the monetary sector relationships were considered at the highest level of aggregation (i.e., money supply and money demand), considered alternative assumptions that qualify for diverse IS and LM slopes and positions. The second set of frameworks, where the monetary sector relationships are disaggregated, also considered different assumptions , that resulted in different implications for the slope and location of the LM curve. In the first part of the chapter we reviewed the analysis performed by experts in the subject of monetary procedures, and it was evident that the importance of the IS and LM slopes has been stressed by many studies. However, we need to analyze in detail the characteristics of the different behavioral relationships that may be essential in determining such different slopes and that, therefore, will prove to be key relationships in the analysis of monetary policy procedures.

The disaggregation of the monetary sector allows for a detailed examination of sources of LM steepness or flatness, that will let us define monetary policy procedures in terms of variables other than the money supply and the interest rate (e.g., reserves, monetary base, etc.).

Chapter 3 will analyze the effect of different shocks (IS and LM) under different monetary policy procedures, for the "aggregated models" described in the present chapter. Later, the same type of analysis will be carried out in Chapter 4, but using the "disaggregated models" as previously described. In this latter case, due to the disaggregation of the monetary sector, we will be able to perform the analysis under policy procedures that will not be considered in Chapter 3 (e.g., using unborrowed reserves as the instrument, etc.).

CHAPTER 3

THEORETICAL ANALYSIS

OF THE AGGREGATED MODELS

INTRODUCTION

Chapter 2 reviewed several recent studies dealing with the problem of choosing monetary policy instruments, and described the theoretical framework used on each of them. Then, it specified the framework to be used in the present study. Several models, corresponding to different structural assumptions, have been proposed and these are summarized in Table 3-1.

The "aggregated" models (hereinafter some times refered to simply as "A"s) do not embody the different elements that determine money supply (i.e., sources and uses of reserves), but rather take money supply only at its last stage (currency plus demand deposits). On the other hand, the "disaggregated" models (hereinafter refered to simply as "D"s) include the elements that make up the money supply from

Table 3-1. Proposed Aggregated and Disaggregated Models: Main Features

Model (identification)	Characteristics			
	Monetary Sector		Real Sector	
	Money supply	Money demand	Investment	Consumption
A-1	Exogenous	Function of income and interest rates	Function of interest rates	Function of income
A-2	Endogenous Function of interest rate and random term	Same as A-1	Same as A-1	Same as A-1
A-3	Same as A-1	Same as A-1	Interest-inelastic	Same as A-1
A-4	Same as A-1	Same as A-1	Fisher effect: investment a function of real interest rates (nominal minus price expectations)	Same as A-1
A-5	Same as A-1	Same as A-1	Same as A-1	Wealth effect: function of wealth
D-1	Disaggregated, exogenous	Demand for currency= demand for deposits, functions of income and interest rates.	Same as A-1	Same as A-1
D-2	Disaggregated, exogenous	Demand for currency and for deposits interest-inelastic	Same as A-1	Same as A-1
D-3	Disaggregated Borrowings and excess Reserves interest-elastic	Same as D-1	Same as A-1	Same as A-1
D-4	Same as D-1	Demand for currency and Dem for deposits as functions of income and interest rates. Inclusion of interest bearing demand deposits	Same as A-1	Same as A-1
D-5	Same as D-3	Same as D-4	Same as A-1	Same as A-1

Source: Chapter 2.

the sources of the monetary base. The initial specification of the models (i.e., A.1 and D.1 respectively) is changed in order to consider alternative assumptions with respect to some of the behavioral relationships in the model's structure (e.g., borrowings as function of interest rates, etc.).

The purpose of this chapter is to specify and analyze, for each of the "aggregated" models, the implications of using either interest rates or a monetary aggregate as a monetary policy instrument. This is done under the assumption that the purpose of monetary policy is to stabilize income; therefore, policy is designed so as to minimize the deviation of income (Y) from its target value(Y*).[64] For each model specification, as characterized by its respective IS and LM equations, we will proceed to do the following analysis for each of the suggested policies:

1. The Policy Procedure.- This refers to the way in which the value of the instrument is determined given the objective function, the assumed economic structure, and the instrument to be used.
2. Implications of Policy Procedure.- This refers to the implications of the specific procedure for the IS-LM equations, and for the endogeneity or exogeneity of the key variables.

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64. The procedure by which authorities determine Y* will not be discussed, but we assume that it is a level considered by them as a full employment level.

3. Reduced Form of Income.- It tells us, given the type of instrument being used in each case, what factors (disturbances, constants and parameters) determine income. Deviations of income from the target will arise if a disturbance appearing in the reduced form is not zero, or if a constant or a parameter in such equation changes during the policy period and thus differs from the one assumed by authorities when setting the instrument value.

Then we will include a section that will summarize the results obtained, specifically with respect to the effects of shocks or disturbances to the economic system under each policy procedure. This section, called "Analytical Simulation of Shocks" will concentrate on:

1. The effect of each type of shock under the same policy procedure but different economic models. This will give an overall view of the previous analysis, highlighting the importance of the different structural assumptions.
2. The effect of each type of shock under the same model but different policy procedures. Given a particular economic structure, this section will highlight the implications of using alternative policies.

Besides the specific assumptions that characterize each model version (as described in Table 3-1), the following assumptions will be kept along the theoretical analysis:

1. Policy makers know the model (coefficients and constants) as specified on each version (unless otherwise specified in a particular simulation problem), and use it in order to determine the value of the instrument.

2. Variables used as instruments are observable during the period for which a value was established. That is, given the ultimate target value, the objective function, the economic

structure and the chosen instruments, the authorities establish a value at which the instrument must be kept for next period. During such period, authorities will use policy tools to maintain it at the pre-set value, hence they will be monitoring the instrument during the policy period. Table 3-2 describes the elements and stages in the formulation of monetary policy. This table describes the "steps" or "rules" that will be assumed as the framework for policy making in the forecoming analysis.

3. The value of the instrument is revised only at the end of a long period (which includes several short periods), unless otherwise specified in a particular case. Such revision is necessary to assure the consistency of the instrument value with the achievement of the ultimate objective (income target).
4. To determine the instrument value, authorities assume that disturbances are zero.
5. When disturbance shocks are simulated, it is assumed that the model's structure (i.e., its coefficients) has not changed, unless otherwise specified in the particular case. Similarly, when parameter changes are simulated, zero disturbances will be assumed.

Table 3-2. Formulation of Short-run Monetary Policy

Steps	Period	Procedure
1	Beginning of <u>i</u> (i=long term)	Determination of income Target Y^*
2	Beginning of <u>i</u> and <u>j</u> (j=short-term) i=1 j=1	Given Y^*, objective function: Min $(Y-Y^*)$, and economic structure as perceived at period <u>i</u>; Determination of instrument value for each period j : I^*_{ij} V_j
3	During i, <u>j</u> (V_j)	Given I^*_{ij}, use policy tools to minimize $(I_{ij}-I^*_{ij})$.
4	Beginning <u>i+1</u>, and j, i=2, j=1	Given Y^*, Objective function: Min $(Y-Y^*)$, and economic structure as perceived at period <u>i+1</u> Determine $I^*_{i+1,j}$ V_j
5	During i+1,j V_j j=1,...,n	Given $I^*_{i+1,j}$, use policy tools to minimize $(I_{i+1,j} - I^*_{i+1,j})$

NOTE: Two period lengths are considered: the long term (i.e., <u>i</u>'s) and the short term (i.e., <u>j</u>'s). Each long-term contains a number "n" of short terms. Monetary policy is designed so as to achieve the i^{th} period's income target Y^*_i, by maintaining (or trying to maintain) the instrument I at a pre-assigned value during each short-term within that long term period.

Finally, Table 3-2 shows the steps within the process of designing and implementing monetary policy. It encloses two different time spans: the first one is longer term planning in which the desired ultimate target is determined and the corresponding value of the instrument is established (rows 1 and 2). Such instrument value is to be maintained during the short-term periods; therefore short-run policy consists in keeping the instrument at the pre-set values (row 3). At the end of the long-term period, the instrument value is revised or adjusted for changes in the system's structure, such that its value is consistent with the achievement of the ultimate income target (row 4). This process continues with a clear-cut division between short and long run responsibilities for monetary policy. In the short-run, monetary policy will try to control the instrument (i.e., keep the instrument value), while the long-run goal is to attain the ultimate target and therefore the relationship between the instrument value and such target is revised.

An alternative to the above scheme will be to assume that the instrument value can be revised sooner (i.e., before the end of the long-term period), in which case I_{ij} will not be the same for all j; step 2 in Table 3-2 would be repeated during some or all of the short-term periods.

I. IMPLICATIONS OF EACH POLICY PROCEDURE

A. MODEL A-1

This is the simplest aggregated version of the models to be studied. It is described mathematically in Table 3-3.

1. Policy Procedure: Interest Rate Instrument

Determination of the Instrument Value

Assuming that Y = Y* is the target level of Income, the value of the instrument (r) is determined by substituting Y* in the IS and LM equations and solving them simultaneously for r. Due to the model's structure, the instrument value for r (r*) turns out to be determined by the IS alone as follows:

$$Y^* = \alpha + \beta r$$

Hence: $$r^* = \frac{(Y^* - \alpha)}{\beta}$$

Where: $U_{is} = 0$ is assumed.

Then, given the instrument value r^* authorities will modify the money supply to keep r at r^*. A diagrammatic description of this procedure is shown in Figure 3-1 .

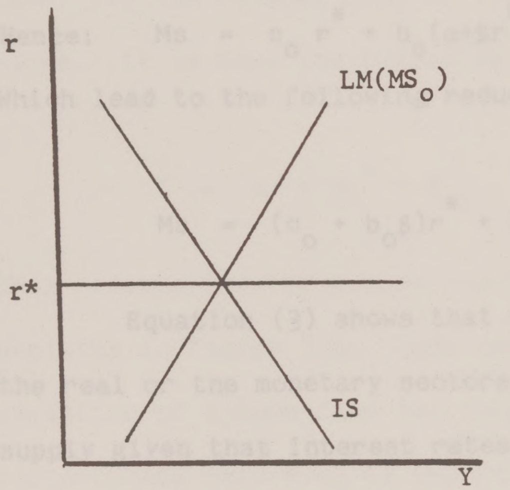
Table 3-3. Model A-1: Structural Equations and Resulting IS-LM

Equations	$Y = C + I + G$ $C = a_1 + b_1 Y + Uc$ $I = a_2 + c_1 r + Ui$ $G = \overline{G}$ $Md = c_0 r + b_0 Y + Umd$ $Ms = Md$
IS	$Y = \alpha + \beta r + Uis$
LM	$Y = \frac{1}{b_0} [Ms - c_0 r - Umd]$
Where:	i) Fixed prices assumed ii) Y = income, c = consumption I = investment, G - government expenditure, r = interest rate; Uc, Ui, Umd are disturbance terms; Md = demand for money; Ms = money supply iii) $\alpha = \frac{\overline{G}+a_1+a_2}{1-b_1} > 0$ $\beta = \frac{c_1}{1-b_1} < 0$

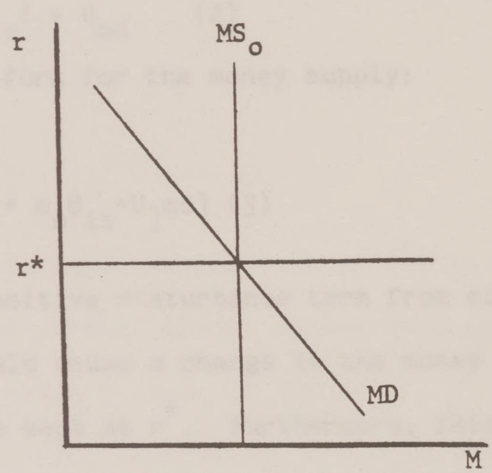
$U_{IS} = \frac{Uc + Ui}{1-b_1}$ $b_1>0, c_1<0, b_o>0, c_0<0$

Figure 3-1

Model A-1. Interest Rate Instrument:
Description of Policy Procedure with
An Interest Rate Instrument.



A. Determination of interest rate instrument value (r^*, given y^*.



B. Monetary sector adjustment to keep $r=r^*$.

Implications of the Procedure

Under this procedure, the money supply becomes a function of the demand for money and of the real sector via income (Y); therefore, the money supply will be endogenously determined as can be shown from the following monetary sector equilibrium condition:

$$Ms \;=\; Md \;=\; c_o r^* + b_o Y + U_{md} \qquad (1)$$

Hence: $$Ms \;=\; c_o r^* + b_o(\alpha+\beta r^* + U_{is}) + U_{md} \qquad (2)$$

Which lead to the following reduced form for the money supply:

$$Ms \;=\; (c_o + b_o\beta)r^* + b_o\alpha + b_o U_{is} + U_{]}md] \quad (3)$$

Equation (3) shows that a positive disturbance term from either the real or the monetary sectors would cause a change in the money supply given that interest rates are kept at r^*. Furthermore, this equation also shows that a change in any of the relevant coefficients (i.e., c_o, b_o, α, β,) will also cause changes in the money supply.

Reduced Form of Income

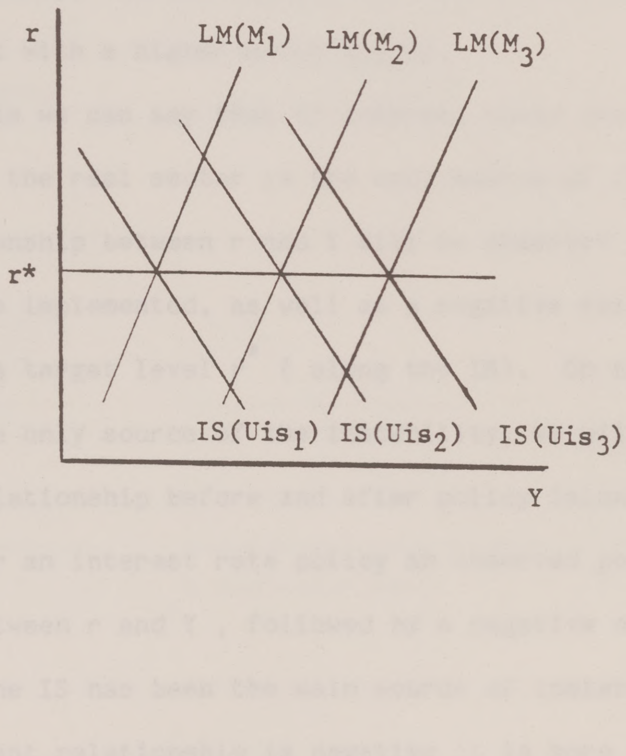
The IS and LM equations embody three unknown variables, two of which can be endogenous and a third which must be exogenously determined in order to have a unique solution to our system. Under the present policy procedure, Income and Money Supply are the endogenous variables, while interest rates have been set exogenously as described above. Given these circumstances, the equilibrium locus of income (that is, the reduced form of income) will be given by the IS equation alone. It is thus the following:

$$Y = \alpha + \beta r^{*} + U_{is}$$

This reduced form of income results in a horizontal line at r^{*}, as depicted in figure 3-2. Such reduced form of income states that any deviation of income from its target value Y^{*} will be due to disturbances occurring in the real sector. Similarly, a change in any of the real sector's parameters (i.e., α, β), will cause income to deviate from its target value Y^{*}. It is useful at this point to review the effects of a disturbance (either U_{is} or U_{lm}) so that the policy procedure that determines interest rates can be fully characterized.

Suppose there is a positive disturbance in the real sector which leads to a shift in the IS curve. As income starts to rise, a higher demand for money presses interest rates up; then monetary policy

Figure 3-2
Model A-1. The Reduced Form of Income with an Interest Rate Instrument

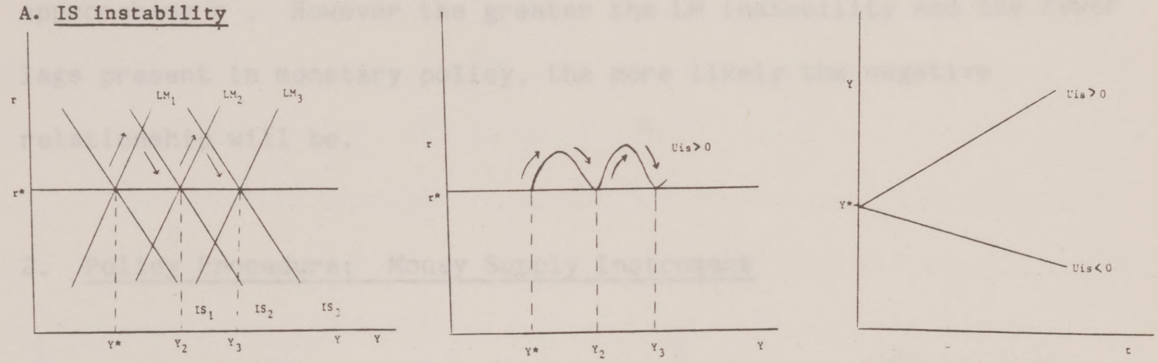


should be such that the monetary authority supplies the additional funds to meet such expanded demand and to bring interest rates down to r^*. The result is the attainment of the pre-established r^*, but with a level of income that is higher than its target value, Y^*. On the other hand, if the source of the disturbance occurs in the monetary sector (e.g., $U_{md}>0$), this will cause income to fall and will press interest rates up above the target r^*; policy then will consist in increasing the money supply in order to offset such pressure and bring interest rates down. Finally, the new equilibrium will be set at the same level of income Y^* but with a higher money supply.

From this we can say that if interest rates are used as the instrument, and the real sector is the only source of instability, a positive relationship between r and Y will be observed (along the LM) before policy is implemented, as well as a negative relationship as r goes back to its target level r^* (along the IS). On the other hand, if the LM is the only source of the instability, we will observe a negative r-Y relationship before and after policy (along IS). Therefore, under an interest rate policy an observed positive relationship between r and Y , followed by a negative one will probably indicate that the IS has been the main source of instability. However, if the predominant relationship is negative it is more plausible to interpret it as a signal of LM instability. Figure 3-3 shows the resulting Y-r relationships under a case of IS instability (A), a case of LM instability(B), and a combination of both(C).

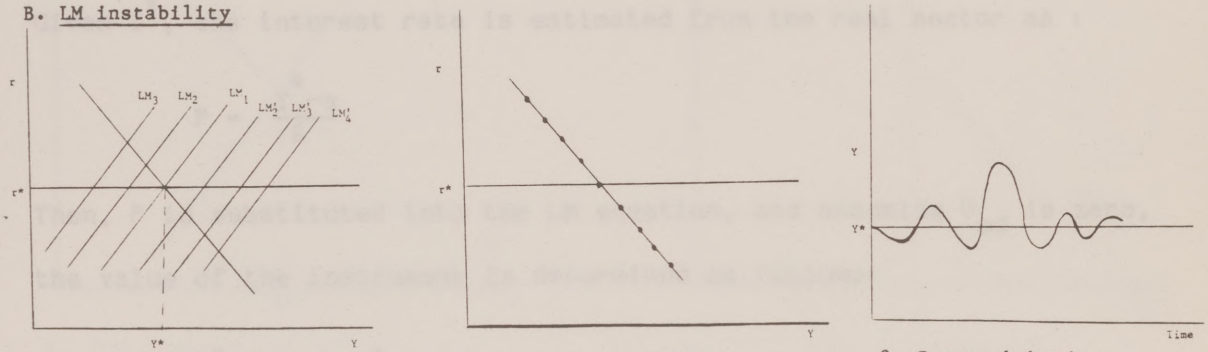
Figure 3-3
Model A-1. Income-Interest Relationship Under
i) IS Instability, ii) LM Instability and
iii) Combinated Instability: With an Interest
Rate Policy Instrument

A. IS instability



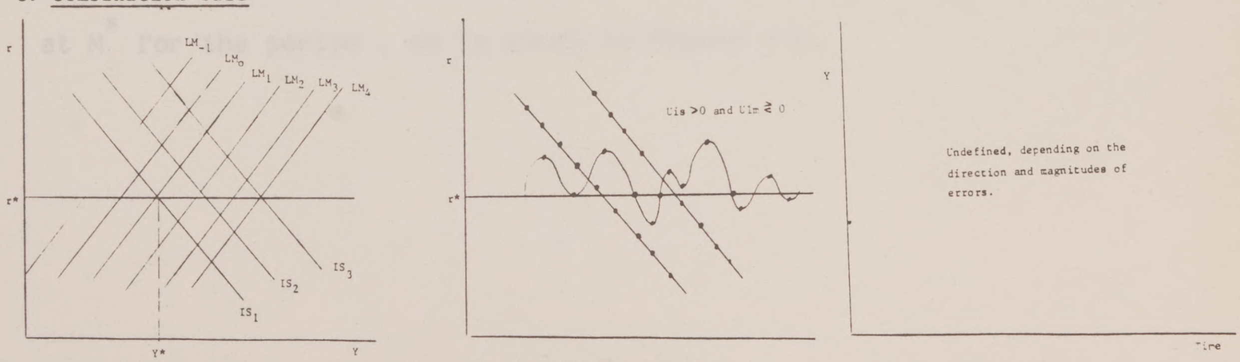
1. IS shifts and effects of policy
2. Equilibrium points
3. Income behavior over time.

B. LM instability



1. LM shifts
2. Equilibrium points
3. Income behavior.

C. Combination Case



1. IS and LM shifts
2. Equilibrium points
3. Income behavior

The last case, where both IS and LM are unstable, reveals that the r-Y relationship cannot be determined a priori. On the one hand, the LM will press the relationship to be a negative one. On the other hand, the IS will press it to be either positive or negative but always approaching r^*. However the greater the LM instability and the fewer lags present in monetary policy, the more likely the negative relationship will be.

2. Policy Procedure: Money Supply Instrument

Determination of Instrument value (M^*)

Given Y^*, the interest rate is estimated from the real sector as :

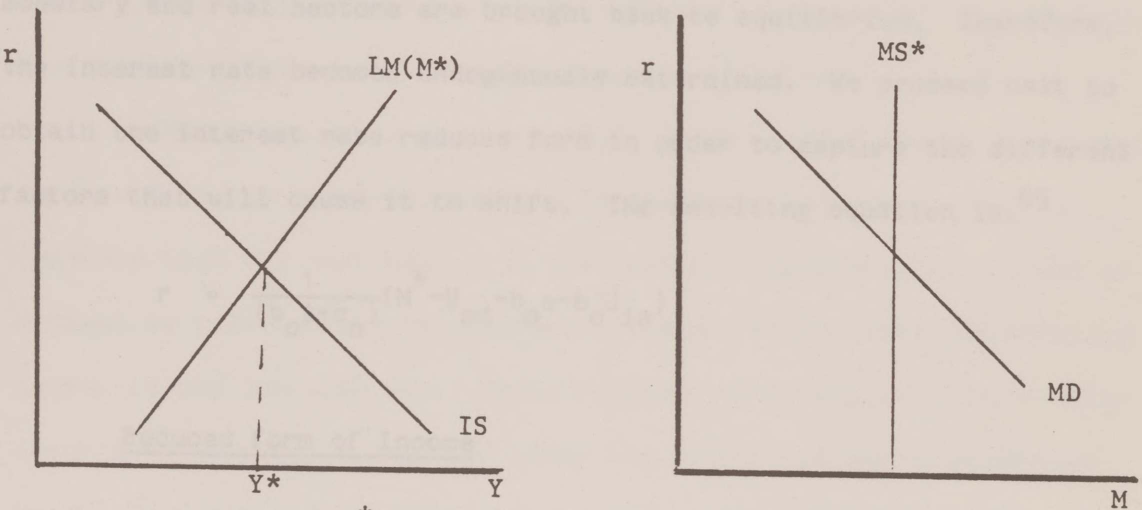
$$\hat{r} = \frac{Y^* - \alpha}{\beta}$$

Then, $\hat{r}$ is substituted into the LM equation, and assuming U_{md} is zero, the value of the instrument is determined as follows:

$$M^* = (b_o Y^* + c_o \hat{r})$$

Under this policy procedure, the money supply is set to remain at M^* for the period , as is shown in Figure 3-4.

Figure 3-4
Model A-1. Description of Policy Procedure
Under a Money Supply Instrument



A. Determination of M^* given Y^*, and IS

B. Adjusting to keep M^*, by fluctuating r.

Implications of the Procedure

In this case, monetary policy will only be concerned with offsetting changes in money supply that cause the money supply to be different from MS^*. Such changes may arise due to changes in borrowings, float, etc., which affect the money supply and are not under direct control of the authorities.

Under a money supply instrument approach, money demand or real sector movements will lead to interest rate changes such that the monetary and real sectors are brought back to equilibrium. Therefore, the interest rate becomes endogenously determined. We proceed next to obtain the interest rate reduced form in order to capture the different factors that will cause it to shift. The resulting equation is:[65]

$$r = \frac{1}{(b_o\beta+c_o)}(M^*-U_{md}-b_o\alpha-b_oU_{is})$$

Reduced form of Income

The IS and LM remain as before, but the equilibrium locus is given by the following expression:

65. For its derivation see the Appendix.

$$Y = \frac{1}{(b_o\beta+c_o)}(\beta M^* + c_o\alpha + c_oU_{is} - \beta U_{md})$$

This reduced form for income shows that under a money supply instrument approach, the level of income will deviate from the target due to disturbances in both the real and the monetary sectors, and also to parameter changes in both sectors.

Given such equation, and because the money supply is fixed at M^*, the equilibrium will occur along the LM if the monetary sector parameters do not change and the disturbance U_{md} is zero. On the other hand, if the real sector's parameters remain unchanged and U_{is} is zero, the equilibrium locus will occur along the IS curve.

This leads us to conclude that, while a money supply instrument is in effect, an observed positive relationship between r and Y may indicate that the real sector is the source of instability, whereas if a negative relationship is observed, it seems likely that the monetary sector is the unstable one. These results, of course, will hold only under a model such as A-1, and under the policy procedure as defined here. From the reduced forms of r and Y we can then obtain the derivatives with respect to each type of disturbance as follows:

$$\frac{\partial r}{\partial U_{md}} = -\frac{1}{(b_o\beta+c_o)} > 0$$

$$\frac{\partial Y}{\partial U_{md}} = -\beta\,\frac{1}{(b_o\beta+c_o)} < 0$$

$$\frac{\partial r}{\partial U_{is}} = -b_o\frac{1}{(b_o\beta+c_o)} > 0$$

$$\frac{\partial Y}{\partial U_{is}} = c_o\frac{1}{(b_o\beta+c_o)} > 0$$

When both U_{is} and U_{lm} occur simultaneously the results are not straightforward. Table 3-4 summarizes the cases when both U_{md} and U_{is} occur simultaneously, giving the effects of such disturbances on the equilibrium values of income and interest rates. These, of course, were derived from the reduced form equations of income and interest rates, respectively.

From Table 3-4 we have the following results. (1) If $U_{md}>U_{is}$, the relationship between Y and r will always be negative, if and only if $c_o<\beta$, and if $b_o<1$. If c_o is greater than β, U_{md} must be relatively larger than U_{is} in order to render a negative Y-r relationship. Hence, the greater the value of c_o, the larger the LM shift must be relative to the IS shift; and the larger β is, the smaller the differnce between U_{md} and U_{is} needs to be. Furthermore, a larger c_o implies a larger $\frac{\partial Y}{\partial r}$ in the LM (i.e., a flatter LM); a larger β means a larger $\frac{\partial Y}{\partial r}$ in the IS (i.e., a flatter IS). Hence, the flatter the LM is (elastic money demand) and the steeper the IS is (less elastic investment and smaller propensity to consume), the larger the monetary sector's shock or

Table 3-4. Model A-1: IS and LM Instability, Their Effects on Income and Interest Rates under a Money Supply Instrument

Source of instability	Effect on Interest Rate (r) (reduced form)	Effect on Income (Y) (reduced form)	Resulting Y-r relationship
$Umd>0$ $U_{IS}>0$	$dr>0$	If $Umd > \frac{co}{\beta}U_{IS}$ $dy<0$	negative
		If $Umd<\frac{co}{\beta}U_{IS}$ $dy>0$	positive
$Umd<0$ $U_{IS}<0$	$dr<0$	If $Umd>\frac{co}{\beta}U_{IS}$ $dy>0$	negative
		If $Umd<\frac{co}{\beta}U_{IS}$ $dy<0$	positive
$Umd<0$ $U_{IS}>0$	If $\lvert Umd\rvert>b_o U_{IS}$ $dr<0$	$dy>0$	negative
(expansive both)	If $\lvert Umd\rvert<boU_{IS}$ $dr>0$		positive
$Umd>0$ $U_{IS}<0$	If $Umd>b_o\lvert U_{IS}\rvert$ $dr>0$	$dy<0$	negative
(recessive both)	If $Umd<b_o\lvert U_{IS}\rvert$ $dr<0$		positive

Source: Appendix.

disturbance must be relative to the real sector's one, for the negative Y-r relationship to hold. (2) If $U_{is} > U_{lm}$, a relationship between Y and r will be positive only if $c_o > \beta$. This final conclusion is a corollary of the above results.

B. MODEL A-2

The additional assumption of this model as compared to A-1 is that now the money supply is a function of both, interest rates and a random term. The complete model specification appears in Table 3-5. It is important to notice that the IS relationship remains as in Model A-1, but the LM has now changed as follows:

$$Y = \frac{1}{b_o}(c_3 r - c_o r + a_3 + U_{ms} - U_{md})$$

The slope of the LM curve is greater than in model A-1 due to the sensitivity of the money supply to interest rates. The slope is:

$$\frac{\partial Y}{\partial r} = \frac{(c_3 - c_o)}{b_o}$$

where c_o is <0 and c_3 is >0.

While in Model A-1 it was:

$$\frac{\partial Y}{\partial r} = \frac{-c_o}{b_o}$$

The constant (a_3) of the money supply function represents the "autonomous" or "exogenous" part of the money supply through which monetary policy can still affect money supply. In other words, it is the component of the money supply that is subject to control by the monetary authorities.

Table 3-5. Model A-2: Structural Equations and Resulting IS and LM

Equations

$$Y = C + I + G$$
$$C = a_1 + b_1 Y + Uc$$
$$I = a_2 + c_1 r + Ui$$
$$G = \overline{G}$$
$$Md = c_0 r + b_0 Y + Umd$$
$$Ms = a_3 + c_3 r + Ums$$
$$Ms = Md$$

IS $\quad Y = \alpha + \beta r + U_{IS}$

LM $\quad Y = \frac{1}{b_0}\left[(c_3 - c_0)r + a_3 + Ums - Umd\right]$

where:

i) Fixed prices are assumed

ii) Y = income; C = consumption; I = investment; G = government expenditure; r = interest rate; Uc, Ui, Umd, Ums = disturbance terms; Md = money demand Ms = money supply; a_3 = autonomous or controllable portion of the money supply

III) $\alpha = \frac{\overline{G} + a_1 + a_2}{1 - b_1} > 0 \qquad \beta = \frac{c_1}{1 - b_1} < 0$

$U_{IS} = \frac{Uc + Ui}{1 - b_1} \quad \frac{c_3 - Co}{b_0} > 0$

$c_3, a_3 > 0 \quad Co < 0 \quad Ulm = \frac{Ums - Umd}{b_0}$

1. <u>Policy Procedure: Interest Rate Instrument</u>

<u>Determination of the Instrument Value</u>

Since Y^* is the income target, the interest rate that will be consistent with such income is given, as in model A-1, by the expression:

$$r^* = \frac{Y^* - \alpha}{\beta}$$

Where: U_{is} assumed $= 0$

α and β known.

Within the procedure of maintaining that interest rate, authorities will use monetary tools (i.e., open market operations) that will certainly affect the "exogenous " element of the money supply (i.e., a_3) and thus the money supply itself. Figure 3-5 shows that the position of the MS schedule is determined by changes in a_3.

<u>Implications of the Procedure</u>

As in Model A-1, with an interest rate instrument r^*, the money supply will be subject to changes in the demand for money and in the real sector through Y. In other words, the money supply will be endogenously determined. Its reduced form will be obtained as follows:

From the monetary sector equilibrium (MS = MD),

$$a_3 = (c_o - c_3)r^* + b_oY + (U_{md} - U_{ms})$$

Figure 3-5
Model A-2. Description of the Policy Procedure
Where Interest Rate is Instrument
(main relationships)

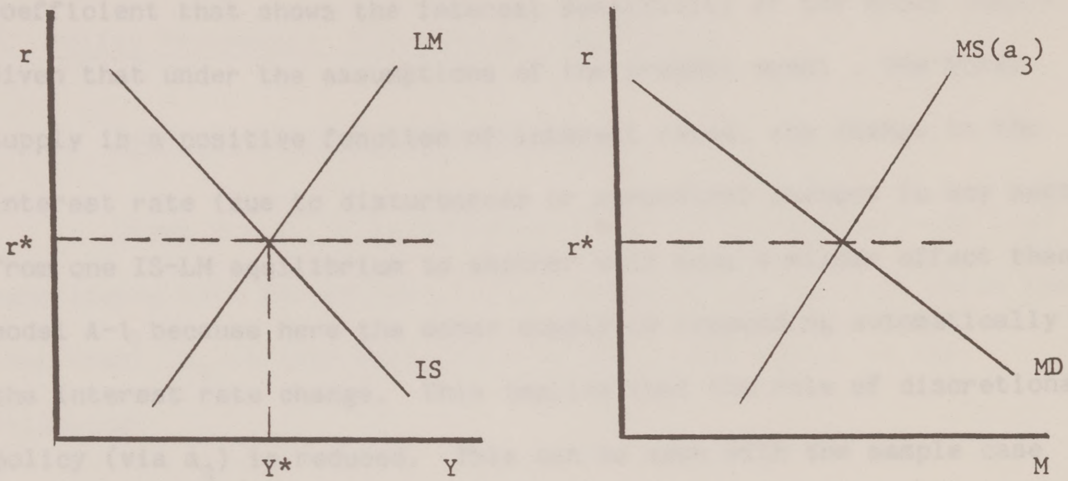
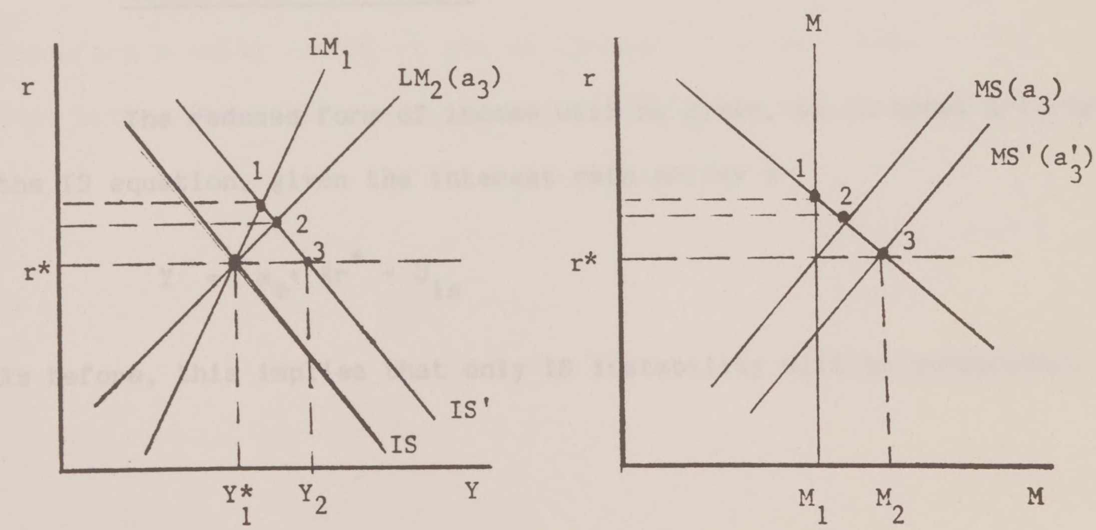


Figure 3-6
Model A-2 Versus A-1 in the Role of Monetary
Policy Under an Interest Rate Policy



Then: $MS = (c_o+b_o\beta)r^* + b_o\alpha + b_oU_{is} + U_{md}$

The difference between a_3 and MS of model A-1 arises from c_3, the coefficient that shows the interest sensitivity of the money supply. Given that under the assumptions of the present model , the money supply is a positive function of interest rates, any change in the interest rate (due to disturbances or structural changes in any sector) from one IS-LM equilibrium to another will have a milder effect than in model A-1 because here the money supply is responding automatically to the interest rate change. This implies that the role of discretionary policy (via a_3) is reduced. This can be seen with the sample case shown in Figure 3-6, it can be seen that the purpose of monetary policy in model A-1 is to bring equilibrium from point 1 to 3 since r^* must be restored. Under A-2 model, monetary policy would have the smaller role of bringing equilibrium from point 2 to 3. In either case, income deviates equally from the target value.

Reduced Form of Income

The reduced form of income will be given, as in model A-1, by the IS equation, given the interest rate policy :

$$Y = \alpha + \beta r^* + U_{is}$$

As before, this implies that only IS instability will be permanently reflected in the equilibrium level of income if interest rate is the instrument.

2. Policy Procedure: Money Supply Instrument

Determination of Instrument Value

Given $Y = Y^*$, r is estimated from the IS equation, and then substituted into the LM equation to determine the value at which the instrument, a_3 in this case, must be kept. The result is the following:

$$a_3^* = b_o Y^* - (c_3 - c_o) \frac{Y^* - \alpha}{\beta}$$

Given: $r = \bar{r}$ and $Y = Y^*$

From here we will consider two different possibilities:

A. The case in which a_3 is defined and observed during policy period; hence a_3 will be kept at a_3^* during such period.

B. The case in which it is not possible to observe a_3 and therefore a value for MS is set as instrument value. This is the value that authorities will try to maintain during the policy period.

Case A

From the monetary sector equilibrium:

$$Ms = a_3+c_3r+U_{ms} = Md = b_oY+c_or+U_{md}$$

And assuming:

$$E(U_{ms} = E(U_{md} = E(U_{is} = 0$$

$$r = \tilde{r}\ Y = Y^*$$

Then: $$a_3^* = b_oY^*+(c_o-c_3)\tilde{r}$$

Or: $$a_3^* = b_oY^*+(c_o-c_3)\frac{Y^*-\alpha}{\beta}$$

Then, under this assumption the money supply will be endogenously determined by:

$$MS = a_3^* +C_3r+U_{ms}$$

which shows the money supply being affected by money supply shocks others than those affecting a_3, and by interest rates, the latter being affected by U_{md} and real sector's shocks.

Case B

If it is not possible to observe a_3(the portion of MS not depending on interest rates) during the policy period, then the money supply is used as the instrument and its value is determined as follows. From the monetary equilibrium and with the same assumptions as in case A, the value of MS^* is derived when equated with money demand:

$$MS^* = b_o Y^* + c_o \bar{r}$$

The instrument value in this case is determined by the demand for money equation.

Under this assumption a_3 becomes endogenous, since it is being determined within the process of attaining Ms*.

From the money supply equation:

$$MS^* = a_3 + c_3 r - U_{ms}$$

But since MS^* is being determined according to the demand, we have:

$$a_3 + c_3 r + U_{ms} = b_o Y^* + c_o \bar{r}$$

$$a_3 = b_o Y^* + c_o \bar{r} - U_{ms} - c_3 r$$

Hence, a_3 will respond to the existence of disturbances and to changes in interest rates (i.e., to $c_3 r$), which are in turn affected by the disturbance of the demand for money equation (U_{md}) and by unanticipated changes in the real sector. The reduced form of the interest rate will tell us precisely what disturbances and other elements affect r and therefore a_3.

Implications of the Procedure

The Interest rate will be the variable that will clear the monetary sector, given a money aggregate target. Therefore interest rates will be endogenous together with Y, while a_3 (Case A above) and Ms (case B) will be the exogenously determined instruments. We now proceed to obtain the reduced form for interest rates under each case.

Case A

Given the IS and LM equations, the IS is substituted into the LM to obtain the reduced form equation for r:

$$\text{IS: } Y = \alpha + \beta r + U_{is}$$

$$\text{LM: } Y = \frac{1}{b_o}((c_3 - c_o)r + a_3^* + U_{ms} - U_{md})$$

Hence:

$$r = \frac{1}{b_o\beta - c_3 + c_o}(a_3^* + U_{ms} - U_{md} - b_o\alpha - b_oU_{is})$$

Case B

In this case we would have:

$$\text{IS: } Y = \alpha + \beta r + U_{is}$$

$$\text{LM: } Y = \frac{1}{b_o}(Ms^* - c_or - U_{md})$$

Hence:

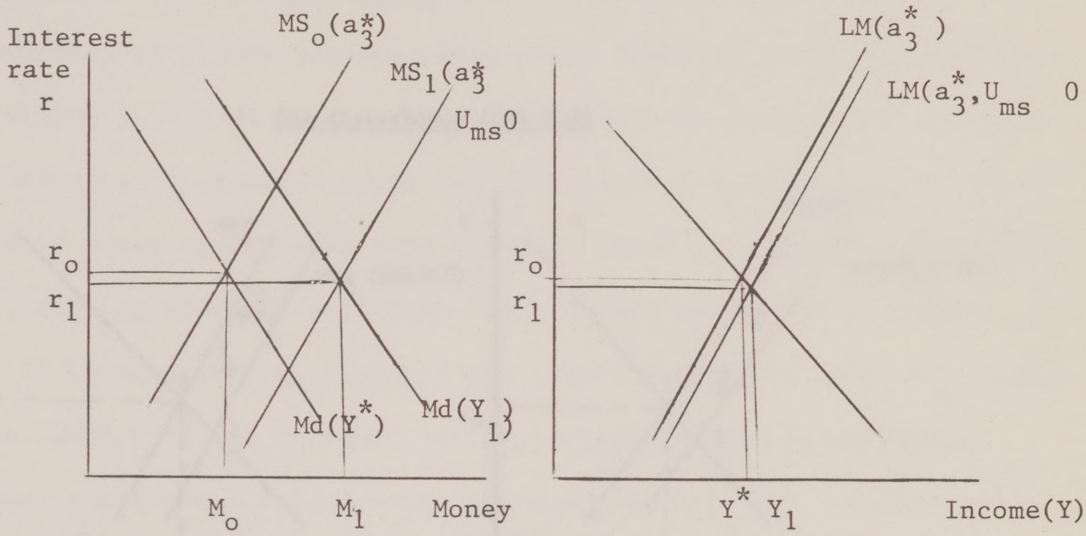
$$r = \frac{1}{c_o + b_o\beta}(Ms^* - U_{md} - b_o\alpha - b_oU_{is})$$

A graphical interpretation of the implications on each case is presented in Figures 3-7 and 3-8.

Figure 3-7 shows the effect of a disturbance on the money supply under

Figure 3-7

Model A-2, Case A: a_3^* used as Instrument.



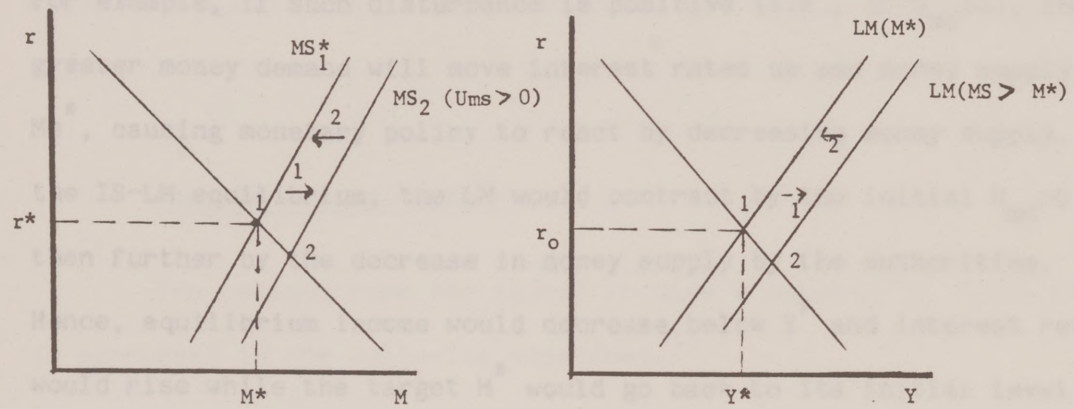
A.1 Equilibrium of Money Demand and Supply

A.2 IS-LM Equilibrium

case A. Since a_3^* has been set, such disturbance will cause a higher money supply together with lower interest rates and income above the target Y^*. This is so since under this policy procedure monetary policy does not react to the disturbance as such disturbance does not affect a_3. Similarly we could simulate the effect of disturbances on the demand for money and on the IS, and the results would also be that interest rates are affected.

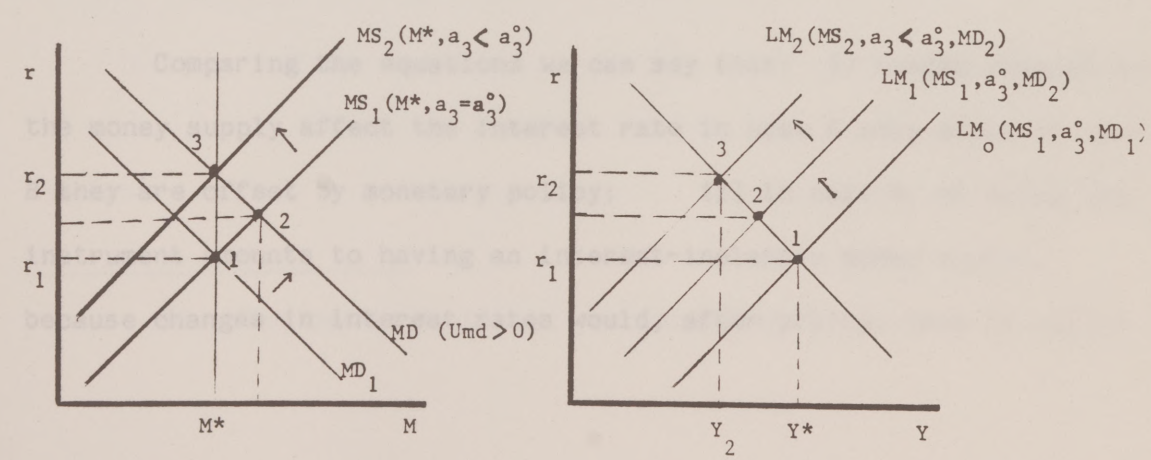
Figure 3-8
Model A-2, Case B (Ms^* as Instrument): Effects of Ums and Umd Disturbances

1. <u>Ums disturbance (Ums > 0)</u>



A. Monetary sector equilibrium B. IS-LM equilibrium

2. <u>Umd disturbance (Umd > 0)</u>



In Case B, shown in Figure 3-8, a disturbance on the money supply (i.e., $U_{ms}>0$) would be offset by the monetary procedure under analysis here. However, a disturbance on the demand for money (simulated in figure 3-8 too) will cause monetary policy to work in a direction that will accentuate the expansion or contraction of income. For example, if such disturbance is positive (i.e., if $U_{md}>0$), the greater money demand will move interest rates up and money supply above Ms^*, causing monetary policy to react by decreasing money supply. On the IS-LM equilibrium, the LM would contract by the initial $U_{md}>0$ and then further by the decrease in money supply by the authorities. Hence, equilibrium income would decrease below Y^* and interest rates would rise while the target M^* would go back to its initial level.

Summarizing the implications of Cases A and B, we have the respective reduced forms of interest rate as:

$$\text{Case A:} \quad r = \xi_a(a_3^* + U_{ms} - U_{md} - b_o\alpha - b_oU_{is})$$

$$\text{Case B:} \quad r = \xi_b(Ms^* - U_{md} - b_o\alpha - b_oU_{is})$$

where: $\xi_a, \xi_b < 0$ but $|\xi_a| < |\xi_b|$

Comparing the equations we can say that: i) random changes in the money supply affect the interest rate in case A only since in case B they are offset by monetary policy; ii) in case B, MS being the instrument amounts to having an interest-inelastic money supply, because changes in interest rates would, after policy, have no impact on the money supply; iii) Both ξ_1 and ξ_b are negative, but the absolute value of ξ_a is smaller than that of ξ_b, which implies that the effect of the disturbances U_{md} and U_{is} on interest rates is greater in case B, in which case monetary policy will react to the fact that Money supply has changed due to those disturbances. Hence the effect on interest rates is enlarged; while the reaction of monetary policy in case A is nil.

Reduced form of Income

The reduced form for income in Case A and Case B, respectively, is expressed in the following equations:[66]

66. For the derivations, see Appendix.

Case A: $Y = \xi_a(\beta a_3^* - (c_3 - c_o)\alpha - (c_3 - c_o)U_{is} + \beta(U_{ms} - U_{md}))$

Case B: $Y = \xi_b(\beta Ms^* + c_o\alpha + c_oU_{is} - \beta U_{md})$

Where: $\xi_a = \dfrac{1}{\beta b_o - c_3 + c_o}$, and $\xi_b = \dfrac{1}{c_o + b_o\beta}$

$$| \xi_a | < | \xi_b |$$

$$\frac{\partial Y_a}{\partial U_{is}} = \frac{c_o - c_3}{b_o\beta - c_3 + c_o} > \frac{\partial Y_b}{\partial U_{is}} = \frac{c_o}{\beta b_o + c_o}$$

$$\frac{\partial Y_a}{\partial U_{md}} = \xi_a(-\beta U_{md})$$

$$\frac{\partial Y_b}{\partial U_{md}} = \xi_b(-\beta U_{md})$$

And since $| \xi_b | > | \xi_a |$

Hence: $\left| \dfrac{\partial Y_b}{\partial U_{md}} \right| > \left| \dfrac{\partial Y_a}{\partial U_{md}} \right|$

In sum, when a monetary aggregate is the chosen instrument, we can only say that a money supply disturbance will cause deviations from target income and cause interest rate movements only under case A, while a money demand disturbance will cause greater deviations from target income as well as greater interest rate fluctuations in case B. In addition, a disturbance originating in the real sector (i.e., U_{is}) will cause greater deviations from target income under case A, but greater interest rate fluctuations under case B.

C. MODEL A-3: Investment Interest Inelastic.

This model will consider a situation in which the determinants of investment are variables other than the interest rate, such as sales forecasts, credit availability, etc. As will become evident, the model delivers an interest-inelastic IS curve, that is, a vertical IS, the real sector thus becoming the sole determinant of Income. In this case, the monetary sector can only affect the level of interest rates, but not income, unless one of the "new" determinants of Investment were related to the monetary sector.[67] The specification of the model, then, will incorporate an exogenous demand for investment. See Table 3-6.

1. Policy Procedure: Interest Rate Instrument

Determination of Instrument Value

In the models analyzed thus far, the interest rate procedure has consisted in determining the instrument value subject to the target $Y = Y^*$. The interest rate that clears the real sector was found, and monetary policy would be directed towards keeping such rate consistent with Y^*. The system was converted into a recursive system where,

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67. This may be the case, but we will not consider it here. We will stick to the interest inelastic investment.

Table 3-6. Model A-3: Structural Equations and Resulting IS-LM

Equations	$Y = C + I + G$
	$C = a_1 + b_1Y + Uc$
	$I = Io + Ui$
	$G=\overline{G}$
	$Md = c_0r + b_0Y + Umd$
	$Ms = Md$
IS	$Y = \frac{1}{1-b_1}\,[a_1+Uc+Io+\overline{G}+Ui]$
LM	$Y = \frac{1}{b_0}\,[Ms - c_0r - Umd]$
where:	i) Prices assumed fixed
	ii) Y = income; C = consumption; I = investment; Io = autonomous investment level; G = government expenditure; r = interest rate; Uc, Ui, Umd = disturbance terms; Md = money demand; Ms = money supply

although we started with three unknowns(i.e., Y, r and M), the value of Y was set exogenously, then the value of the instrument r was determined by the real sector's IS, whereupon such Y and r values determined the value that M would have in the LM equation. In the present model, however, the value of the interest rate cannot be determined by the real sector; furthermore, Y will be determined by the real sector while monetary policy will have no way to pursue a policy directed to attain a target Y^*. Deviations of income from the desired Y^* are due to the real sector's elements and cannot be prevented through monetary policy. Given the level of income generated in the real sector, any interest rate would be consistent with it. Therefore, it is not possible to set one single value (r^*) that would serve as the instrument value to attain Y^*. The only way to change the level of income under the present circumstances is through fiscal policy; the scope for monetary policy is therefore limited and cannot be used to achieve income stabilization.

It seems appropriate to say then, that under the assumption of a vertical IS, monetary policy can only be used to stabilize interest rates or the money supply. In the first case, r^* would be specified by past experience and so monetary policy would aim to keep r^* through money supply movements (money supply thus becoming endogenous). This is the approach we will follow for our analysis in this section.

Implications of this Procedure

This new approach to monetary policy implies that income is determined by the IS as:

$$Y = \frac{1}{1-b_1} (a_1 + I + G + U_c + U_i)$$

Then, if r^* has been specified, MS becomes endogenously determined with its reduced form being:[68]

$$MS = \frac{b_o}{1-b_1} (a_1 + + I + G + U_c + U_i \quad) + c_o r^* + U_{md}$$

Figure 3-9 describes the IS-LM and the monetary sector equilibrium under the present assumptions, and simulates the effect of a shift (increase) in money demand on money supply, since income remains at the initial level determined by the IS. Movements go from A to B and to C in the monetary sector's diagram, and from A to B and back to A in the IS-LM equilibrium diagram.

The money supply in this case will be subject to disturbances occurring in either the demand side of the monetary sector (i.e., U_{md}) or in the real sector(i.e., Uc or Ui).

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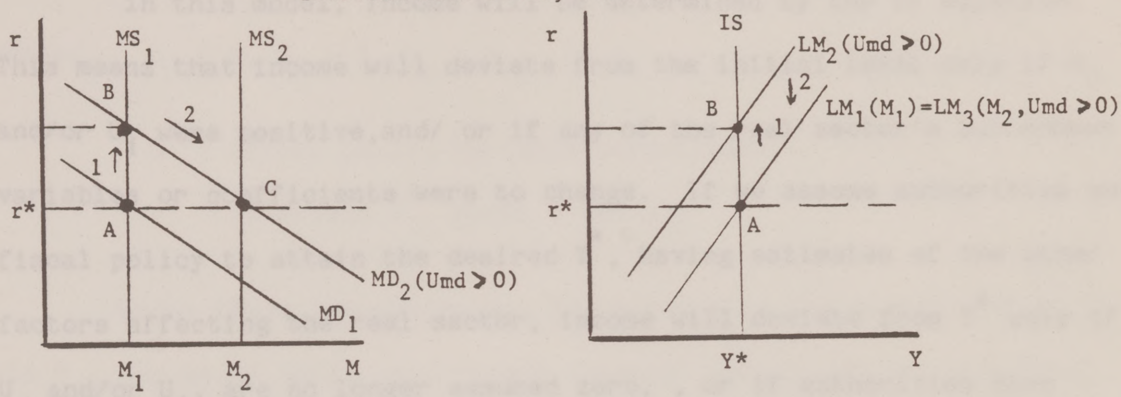
68. For derivation see Appendix.

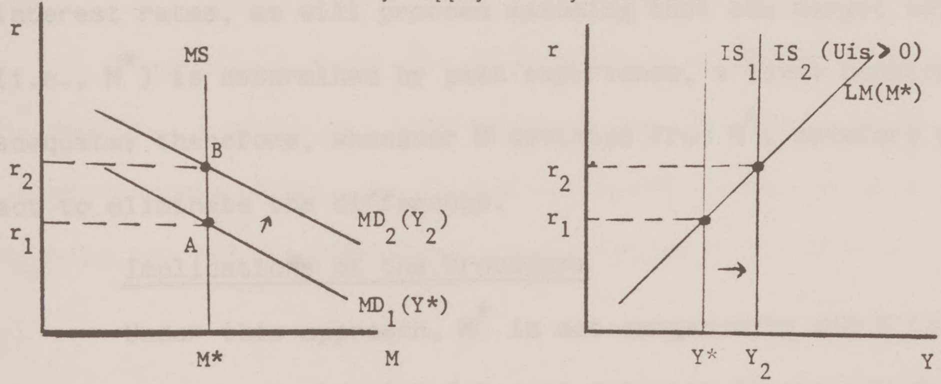
Figure 3-9
Model A-3. Effects of a Money
Demand Disturbance Under an
Interest Rate Policy Procedure



A. Equilibrium of monetary sector

B. IS-LM equilibrium

Figure 3-10
Model A-3. Effects of an IS Disturbance under
A Money Supply Policy Procedure



A. Equilibrium in the monetary sector

B. IS-LM equilibrium

Reduced Form of Income

In this model, income will be determined by the IS equation. This means that income will deviate from the initial level only if U_c and/or U_i were positive,and/ or if any of the real sector's autonomous variables or coefficients were to change. If we assume authorities use fiscal policy to attain the desired Y^*, having estimates of the other factors affecting the real sector, income will deviate from Y^* only if U_c and/or U_i, are no longer assumed zero, , or if authorities have mis-estimated the parameters and constants on the IS equation. However, as was explained before, income is not considered a target for monetary policy under the present framework.

2. Policy Procedure: Money Supply Instrument.

Determination of Instrument Value

If authorities choose to stabilize the money supply instead of interest rates, we will proceed assuming that the target value of M (i.e., M^*) is determined by past experience, a level considerated adequate; therefore, whenever M deviates from M^*, monetary policy will act to eliminate the difference.

Implications of the Procedure

Under this approach, M^* is set exogenously and Y is determined by the real sector, leaving interest rates to be determined endogenously by market forces . Its reduced form is given by the following expression:[69]

$$r = -\frac{b_o}{c_o(1-b_1)}(a_1 + I + G^* + U_c + U_i) + \frac{MS^*}{c_o} - \frac{U_{md}}{c_o}$$

Where: $c_o < 0,\ b_1 > 0,\ b_o > 0$

G^* = fiscal policy variable established to attain Y^*, given I and a_1.

Under these circumstances, an increase in the demand for money via U_{md}, or an IS shift will affect interest rates, with no offseting monetary policy action as a response. The only reason for monetary policy to respond to market forces under the present model would be if the money supply changes as a result of forces outside monetary policy control (e.g., increase in borrowings or excess reserves of banks) or if the money supply is interest-elastic; however, the model specification does not allow explicit consideration of these changes, thus assigning monetary policy a rather passive role. Such considerations will be possible in subsequent specifications of the model.

Figure 3-10 shows the effects of an IS shift under the present MS^* policy. A positive disturbance in the real sector would lead to higher interest rates, due to the increased demand for real balances which is not met by additional supply (since M^* is fixed).

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69. See derivations in Appendix.

Reduced Form of Income

The reduced form of income in this case is again given by the real sector, that is, by the IS equation.

Concluding from the above two cases we can say that, under a policy of maintaining r^*, a shock to the LM via U_{md} will bring temporary interest rate movements which, after policy reaction, will drive interest rates back to their target level. On the other hand, if the policy was to keep M^*, interest rates would have changed and no policy reaction would have occured. In both cases, income remains at its initial level since the real sector has suffered no change.

If the real sector's coefficients or disturbances change, both interest rates and income will be affected initially. Then, under the r^* policy, monetary policy will be expansive if the real sectors' change was expansive and it will be restrictive if the real sectors' change was also restrictive. Under the M^* policy, on the other hand, the changes in Y and r due to the real sector's behavior will remain unchanged since no monetary policy action will arise.

These results lead us to conclude that the instrument choice problem for monetary policy under the A-3 model assumptions lie in the policy maker's preference of a stable interest rate over a stable money supply. The role of monetary policy in income stabilization is nil under the assumption that the real sector is not affected by interest rates; therefore, the roles of monetary and fiscal policy are clear cut: monetary policy to pursue either interest rate or money supply stabilization, and fiscal policy to pursue income stabilization.

D. MODEL A-4: FISHER EFFECT ON INVESTMENT

This model, incorporating the effect of price expectations on investment, is specified on Table 3-7.

1. Policy Procedure: Interest Rate Instrument

Determination of Instrument Value

Given $Y = Y^*$ and that authorities estimate price expectations as $\tilde{\dot{P}}e$ and assume disturbances are zero, from the IS equation in Table 3-7, the value of the instrument is obtained as:

$$r^* = \frac{Y^* - \alpha + \beta_2 \tilde{\dot{P}}e}{\beta_2}$$

Where: $$\tilde{\beta}_2 = \frac{\beta}{1+\tilde{\dot{P}}e}$$

This is the value at which authorities will keep the level of interest rates. In this case, as before, the money supply will be changing in the process of trying to attain r^*.

Implications of the Procedure

While establishing an interest value as the short-term focus for policy, money supply becomes the endogenous variable. Its reduced form will be given by the following expression:[70]

Table 3-7. Model A-4: Structural Equations and Resulting IS-LM (Fisher effect included)

Equations	$Y = C + I + G$
	$C = a_1 + b_1 Y + Uc$
	$I = a_2 + c_1 \rho + Ui$
	$\rho = \dfrac{r - \dot{P}e}{1 + \dot{P}e}$
	$\dot{P}e = \text{case A} \sum_{i=1}^{n} Ai\left[\dfrac{Y_{t-i} - Y_{t-i-1}}{Y_{t-i-1}}\right]$
	$\text{case B} \sum_{i=1}^{n} W_i\left[\dfrac{M_{t-i} - M_{t-i-1}}{M_{t-i-1}}\right]$
	$G = \overline{G}$
	$Md = c_0 r + b_0 Y + Umd$
	$Md = Ms$
IS and LM	LM: $Y = \dfrac{1}{b_0}\left(Ms - c_0 r - Umd \right)$
	IS: $Y = \alpha + \beta_2 r - \beta_2 \dot{P}e + U_{IS}$
where:	i) Y = income; C = consumption; I = investment; G = government expenditure; r = nominal interest rate; ρ = expected real interest rate; $\dot{P}e$ = expected rate of inflation
	$\beta_2 = \dfrac{\beta_1}{1+\dot{P}e} = \dfrac{c_1/1-b_1}{1+\dot{P}e}$
	where $\beta_2 < \beta_1$ (see chp. 2)
	Ms = money supply; Md = demand for money; Ui, Uc, Umd = respective disturbance terms.
	ii) β_1 is the IS slope in previous specifications.
	iii) $\dot{P}e$ is a pre-determined variable.

$$MS = (c_o + b_o\beta_2)\, r^* + b_o\alpha - b_o\beta_2\dot{P}e + b_oU_{is} + U_{md}$$

Reduced Form of Income

The reduced form will be given by the IS equation.

$$Y = \alpha + \beta_2 r^* - \beta_2\dot{P}e + U_{is}$$

which indicates that unpredicted changes in price expectations, in α and in β_2, as well as a positive real sector's disturbance will affect income.

However, different assumptions about how price expectations (i.e., $\dot{P}e$) are determined, may render different conclusions with respect to the elements determining income. The following analysis will consider two initial possibilities: one, that price expectations are mainly determined by past rates of income growth; and second, that they are determined by past rates of money growth. Then, within each of those possibilities we will study several cases, as described in Table 3-8. The implications of each case are now considered.

Case A. Price Expectations determined by Income Growth.

The assumptions that will be kept in analyzing this relationship within the complete IS-LM model are: i) prices are not explicitly explained

70. For derivation see Appendix .

Table 3-8. Model A-4: Cases A and B and Alternative Possibilities that Will be Considered

	Case A	Case B
	$\dot{P}e = \sum_{i=1}^{n} a_i \left(\frac{Y_{t-i} - Y_{t-i-1}}{Y_{t-i-1}} \right)$	$\dot{P}e = \sum_{i=1}^{n} W_i \left(\frac{M_{t-i} - M_{t-i-1}}{M_{t-i-1}} \right)$
Policy revised at end of short-term period	$\tilde{\dot{P}}e = \dot{P}e$ $\tilde{\dot{P}}e \neq \dot{P}e$	$\tilde{\dot{P}}e = \dot{P}e$ $\tilde{\dot{P}}e \neq \dot{P}e$
Policy not revised	$\tilde{\dot{P}}e = \dot{P}e$ (when no disturbance occurs) $\dot{P}e \neq \dot{P}e$ (if any disturbance occurs)	$\tilde{\dot{P}}e = \dot{P}e$ (if no disturbance occurs) $\dot{P}e \neq \dot{P}e$ (if any disturbance occurs)

where:

$\tilde{\dot{P}}e$ = price expectation as estimated by authorities.

$\dot{P}e$ = price expectations as true.

Y_{t-i} = income period t-i.

M_{t-i} = money supply period t-i.

$a_i + w_i$ = coefficients that determine the lag structure.

within the model, ii) price expectations depend upon past changes in income, hence the price expectations is a "pre-determined" variable, defined as follows:

$$\dot{P}e_t = \Sigma a_i \left(\frac{Y_{t-i}-Y_{t-i-1}}{Y_{t-i-1}}\right)$$

iii) authorities take into account the existence of price expectations when setting the value of the monetary policy instrument, using their own estimate of $\dot{P}e$ (hereinafter refered to as $\tilde{\dot{P}}e$). And iv) It will still be assumed that the relevant target variable is Income, without distinguishing between real and nominal income.

Now we proceed to consider further possibilities within this framework:

i)<u>Policy is revised at end of each short-term period.</u>

This means that r^* is being changed as authorities estimate $\dot{P}e$ are changing. In this case we assume authorities estimate price expectations as a function of past income growth. However, the next question is if $\dot{P}e$ is estimated correctly or not.

i.1) <u>Pe estimated correctly .</u>

In this case, $\tilde{\dot{P}}e = \dot{P}e$, and when this assumption is substituted into the reduced form equation for income we obtain the following expression of income in terms of "target income":[71]

$$Y_{ij} = Y^*_{ij}+U_{is}$$

i.2) <u>$\dot{P}e$ estimated incorrectly.</u>

In this case the reduced form of income becomes:

$$Y_{ij} = Y^*_{ij} + \beta_2(\tilde{\dot{P}}e_{ij} - \dot{P}e_{ij}) + U_{is}$$

Where $(\tilde{\dot{P}}e_{ij} - \dot{P}e_{ij})$ is a random, estimation error.

In order to capture the implications of this situation, the case in which $\tilde{\dot{P}}e > \dot{P}e$ is simulated in Figure 3-11. The initial equilibrium is given by IS_1 and LM_1 at Y^* and r^*_1, given initial price expectations $\dot{P}e_1$. These change to $\dot{P}e_2$ although authorities estimate they have changed to $\tilde{\dot{P}}e_2$ which is greater than the true expectations. In the revision process for determining r^*, authorities establish r^*_2 as the instrument value that must be maintained to attain Y^*. However, as the figure indicates, such interest rate level will deliver a level of income equal to Y_2 which is less than Y^*. The problem has been that, due to the incorrect estimation of $\dot{P}e$, authorities have fixed the instrument value too high; hence, monetary policy is used to attain such interest rate level, causing nominal rates to rise more than the increase in true price expectations. However, if we follow the dynamic implications of the present model specification, the fall in income will lower expectations ($\dot{P}_2$ falls to $\dot{P}_3$ in Figure 3-11, Diagram II).

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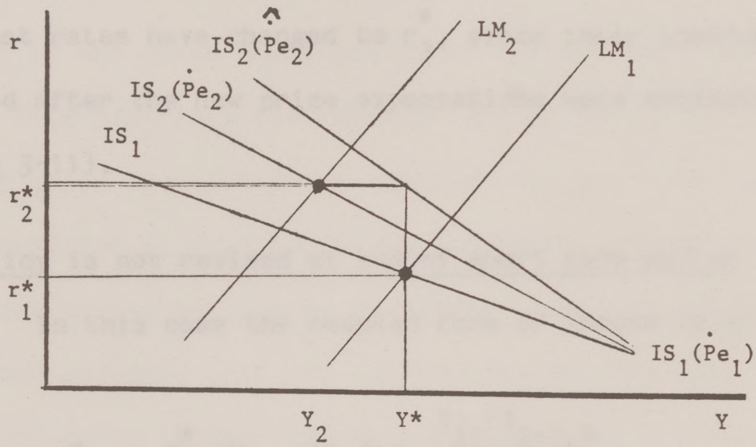
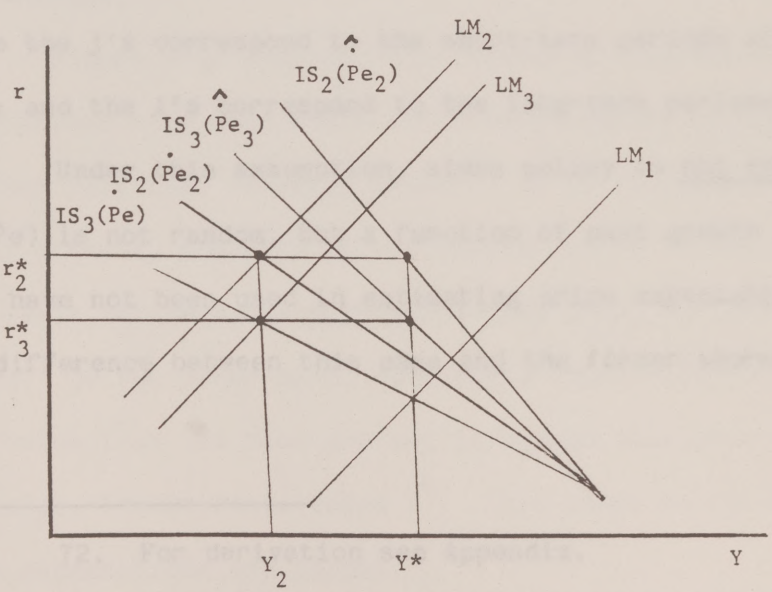
71. This equation assumes that the rest of the parameters (e.g., α, etc.) were correctly estimated by authorities when setting the instrument value. For the derivation see Appendix.

Figure 3-11
Model A-4. Case A, r^* Policy, Revised Every Short Term, and $\tilde{\hat{P}}e > \hat{P}e$



I. Price expectations increase from $\dot{P}e_1$ to $\dot{P}e_2$ and are over estimated ($\hat{P}e > \dot{P}e$.



II. As income flals, price expectations are reduced and policy re-adjusts r^* again. Assumption: authorities correctly estimate the effect of the income fall in $\dot{P}e$.

But if the effect of the fall in income is correctly forecasted, then $\tilde{\dot{P}}e_2$ falls just as the true Pe_2 fell (to $\tilde{\dot{P}}e_3$), such that :

$$(\tilde{\dot{P}}e_2-\dot{P}e_2) = (\tilde{\dot{P}}e_3-\dot{P}e_3)$$

which will imply no further change in income (from Y_2), although interest rates have changed to r_3^*, since their instrumental value was revised after the new price expectations were estimated (Diagram II of Figure 3-11).

ii) Policy is not revised at end of short term period

In this case the reduced form of income is given by:[72]

$$Y_{ij} = Y^*_{ij}+U_{is_{ij}}+\beta_2(-a_{i1}\frac{Y_{i1}-Y_{i-1,4}}{Y_{i-1,4}}$$

$$-a_{i,2}\frac{Y_{i,2}-Y_{i,1}}{Y_{i,1}}-\dots-a_{i,j}\frac{Y_{i,j}-Y_{i,j-1}}{Y_{i,j-1}}) \qquad \text{For } j>1$$

Where the j's correspond to the short-term periods within the long-term ones; and the i's correspond to the long-term periods.

Under this assumption, since policy is not revised, the error $(\tilde{\dot{P}}e-\dot{P}e)$ is not random, but a function of past growth rates of income that have not been used in estimating price expectations. This is the key difference between this case and the former where policy is revised

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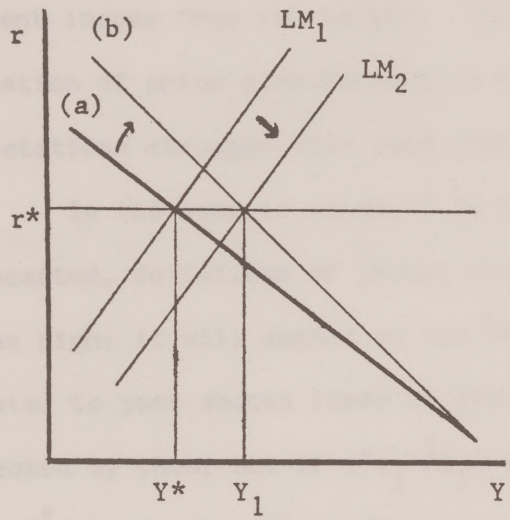
72. For derivation see Appendix.

but $\tilde{\dot{P}}e$ was incorrectly estimated.

In this case, then, Y_{ij} will be affected by changes in income that occurred in past short-term periods(j's) of the present long term period i, and that are themselves affected by U_{is} disturbances that were in effect during such periods. Only when j = 1 the result will be similar to the cases where policy is revised every short-term period. Figure 3-12 describes the implications of the no revision assumption for the cases in which price expectations are under and over estimated by authorities (i.e., when $\dot{P}e > \tilde{\dot{P}}e$ and $\dot{P}e < \tilde{\dot{P}}e$, respectively). These cases are equivalent to the IS shifts caused by disturbances or by parameter changes, with the difference that the effects of price expectations on income will further affect price expectations in following periods. In addition, since there is no revision, there will be a continuous change in income and in money supply to accommodate such changes.

Figure 3-12 suggests the following conclusions. When price expectations rise (and hence are underestimated by authorities), the interest rate that is being used(r^*) is lower than the necessary to keep Y^*; hence, monetary policy results in an income level above Y^*. This leads to greater expectations that press for even higher levels of income. On the other hand, when price expectations fall, the interest rate value that has been pre-set is higher than that necessary to keep Y^*; hence, Income falls below Y^*. This leads to further reductions of price expectations and income.

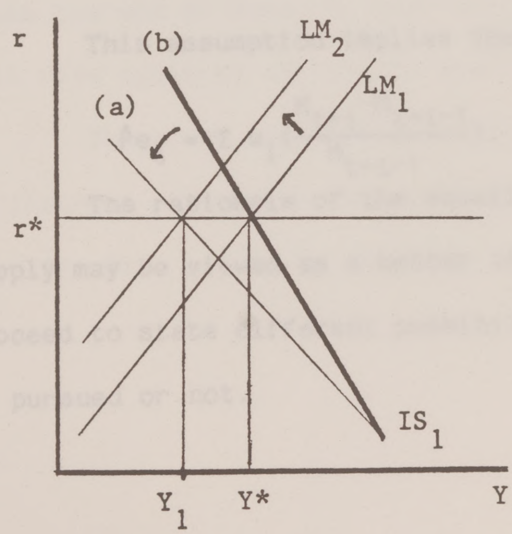
Figure 3-12
Model A-4, Case A, r^* Policy, Not Revised in
Short Run: Effects of a Change in Price Expectations



I. Price expectations increase; and since no revision $\hat{\dot{P}}e > \dot{P}e$.

(a) : $IS_1(\hat{\dot{P}}e_1 = \dot{P}e_1)$

(b) : $IS_2(\dot{P}e_2 > \hat{\dot{P}}e_1)$



II. Price expectations fall; and since no revision, $\dot{P}e < \hat{\dot{P}}e$.

(a) : $IS_2(\dot{P}e_2 < \hat{\dot{P}}e_2)$

(b) : $IS_1(\hat{\dot{P}}e_1 = \dot{P}e_1)$

What this implies is that if the value of the instrument is not revised after each short period, past shocks to the real sector will affect present income; specifically, they will cause deviations of present income from its target. This would be aggravated if the formation of price expectations is such that more recent events affect expectations stronger than less recent ones.

In the case in which r* is revised but is not perfectly forecasted, deviations of income from the target will exist but may not be as high; it will depend on how $\dot{P}e$ are forecasted. If $\dot{P}e_j-\tilde{\dot{P}}e_j$ is related to past shocks (case ii above) , then $(Y_{ij}-Y^*_{ij})$ will be affected by them, but if $(\dot{P}e_j-\tilde{\dot{P}}e_j)$ is random (case i.2 above), then $(Y_{ij}-Y^*_{ij})$ may also be random.

Case B. Price Expectations depend upon past growth rates of Money Supply.

This assumption implies that:

$$\dot{P}e_t = \Sigma\, w_i\left(\frac{M_{t-i}-M_{t-i-1}}{M_{t-i-1}}\right).$$

The rationale of the equation is that growth of the money supply may be viewed as a better indicator of price changes. We now proceed to state different possibilities depending on whether revision is pursued or not.

i) Policy is revised at the end of each period

i.1) $\dot{P}e$ estimated correctly.

In this case, since $\tilde{\dot{P}}e_{ij} = \dot{P}e_{ij}$, the instrument value is given by:

$$r^*_{ij} = \frac{Y^*_{ij} - \alpha + \beta_2 \dot{P}e_{ij}}{\beta_2}$$

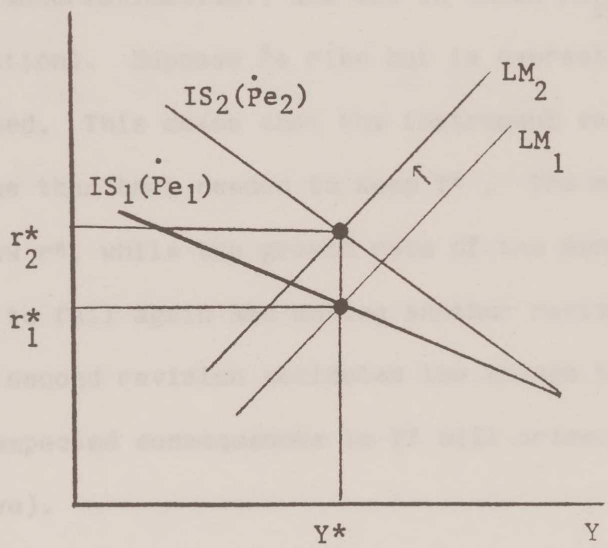
and as in case A above, the reduced form of income is:

$$Y_{ij} = Y^*_{ij} + U_{is_{ij}}$$

However, the value of r^*_{ij} may differ from that in case A above. In Figure 3-13, suppose price expectations rise to $\dot{P}_2$ but they are perfectly predicted by the authorities ($\tilde{\dot{P}}e = \dot{P}e$). Hence, r* is adjusted to keep Y* given the new $\dot{P}e$, rising from r^*_1 to r^*_2. Monetary policy will be directed to attaining r^*_2, which implies a falling money supply (hence LM_1 shifts to LM_2). However, as money supply falls, $\dot{P}e$ falls too and so does r*, with the consequence that the money supply will rise again to attain the lower r* (between r^*_1 and r^*_2).

Therefore, in this case the final equilibrium would be set at an interest rate lower than that of case A above due to the different specification of the $\dot{P}e$ function.

Figure 3-13
Model A-5, Case B, r^* policy, Revised
in the Short Run: Effects of an Increase in
Price Expectations Assuming Authorities
Forecast $\dot{P}e$ Perfectly
(i.e., $\tilde{\dot{P}}e = \dot{P}e$)



i.2) Pe is estimated incorrectly.

In this case $\tilde{\dot{P}}e_{ij} \neq \dot{P}e_{ij}$, with similar results to case A above. The reduced form of income is :

$$Y_{ij} = Y^*_{ij} + \beta_2(\tilde{\dot{P}}e_{ij} - \dot{P}e_{ij}) + U_{is}$$

Where $(\tilde{\dot{P}}e_{ij} - \dot{P}e_{ij})$ is random.

In this case we can distinguish two situations: one in which $\tilde{\dot{P}}e_{ij} < \dot{P}e_{ij}$ (underestimation), and one in which $\tilde{\dot{P}}e_{ij} > \dot{P}e_{ij}$ (overestimation). Suppose $\dot{P}e$ rise but is overestimated ($\tilde{\dot{P}}e > \dot{P}e$) as it is revised. This means that the instrument value r* is set at a higher value than that needed to keep Y* . The money supply falls to keep the new r*, while the growth rate of the money supply decreases causing $\dot{P}e$ to fall again and making another revision necessary. If we assume the second revision estimates the change in $\dot{P}e$ correctly, no further unexpected consequences in Y* will arise.(See explanation in case A above).

The key element that could make the final results here to differ from those in case above A are: i) the relationship that determines $\dot{P}e$ (e.g., M in this case and Y in the case above); ii) the relative changes in money and income after each shock, since these have further effects on $\dot{P}e$; and iii) the lag structure that determines $\dot{P}e$ (e.g., whether more recent values have greater impact than older ones).

ii)<u>Policy is not revised each jth (short-term) period, but only every ith (long term) period</u>.

If j = 1, which means that we are within the first short-term period in the long-term period, the results would be equal to those in which there is revision, with all the different possibilities. If j>1, the situation will change. The results are derived as before (Case A) and so we obtain the reduced form of income as:

$$Y_{ij} = Y^*_{ij}+\beta_2(-w_{i1}(\frac{M_{i1}-M_{i-1,4}}{M_{i-1,4}}-w_{i2}(\frac{M_{i2}-M_{i1}}{M_{i1}})$$

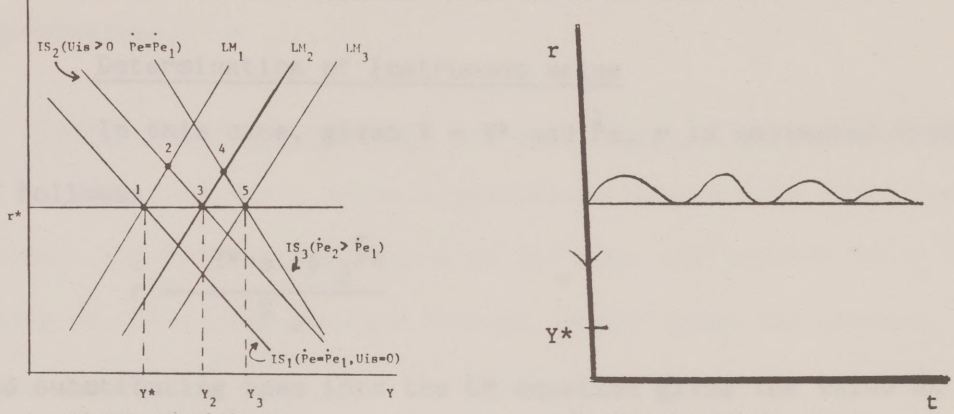
$$-\ldots.-w_{i,j-1}(\frac{M_{i,j-1}-M_{i,j-2}}{M_{i,j-2}})+U_{is_{ij}}$$

However, there is an important difference between these results and those in the case in which expectations depend on past income changes. In the latter,past income changes are affected by past U_{is} errors, while in the present model, the reduced form of income contains past changes in the money supply which are not only affected by U_{is} errors but also by U_{md} errors (see the reduced form of the money supply).

Therefore, under the present assumptions deviations of income from target will be affected by current real sector shocks plus past real sector shocks and monetary sector shocks.

Figures 3-14 and 3-15 show the dynamic effects on income and interest rate behavior, of a real sector's shock and a monetary sector's shock.

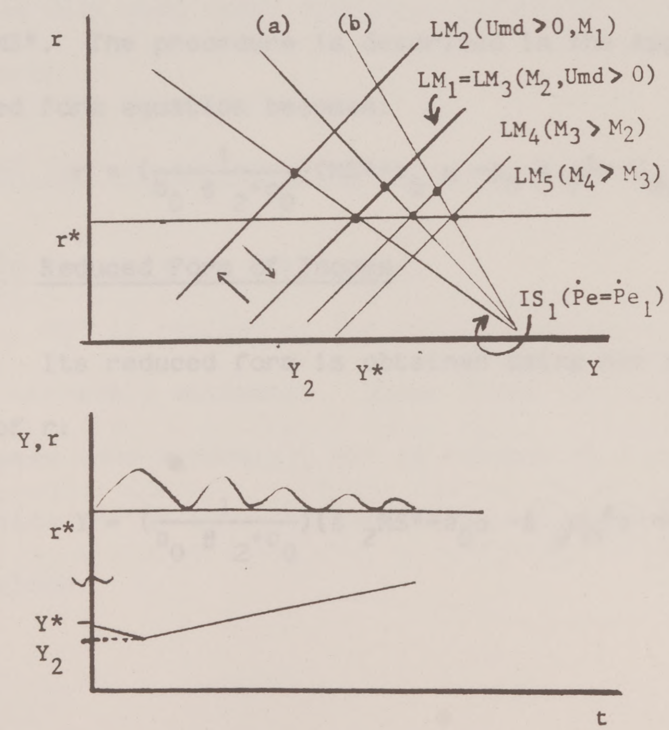
Figure 3-14
Model A-4, Case B, r^* Policy, No Revision in
Short Term Periods: Effect of An IS Shock (Uis > 0)



A. IS-LM equilibrium given initial Uis > 0, and subsequent effects.

B. Income and interest rate behavior given initial Uis > 0.

Figure 3-15
Model A-4, Case B, r^* Policy, No Revision in
Short Term Periods: Effects of An LM Shock (Umd > 0)



A. IS-LM equilibrium: initial LM shift (Umd>0) and subsequent effects

(a): $IS_2(\dot{P}e_2 > \dot{P}e_1)$

(b): $IS_3(\dot{P}e_3 > \dot{P}e_2)$

B. Income and interest rate behavior given initial Umd > 0

2. Policy Procedure: Money Supply Instrument

Determination of Instrument value

In this case, given $Y = Y^*$ and $\tilde{\dot{P}}e$, r is estimated from the IS as follows:

$$\tilde{r} = \frac{Y^*-\alpha + \beta_2 \tilde{\dot{P}}e}{\beta_2}$$

and substituting them into the LM equation gives the value at which the money supply must be kept to attain Y*:

$$MS^* = Y^*(b_0 + \frac{c_0}{\beta_2}) - \frac{c_0 \alpha}{\beta_2} + c_0 \tilde{\dot{P}}e$$

Implications of this Procedure

Under this procedure the interest rate becomes endogenously determined. Its reduced form is obtained by equating LM and IS given $MS = MS^*$. The procedure is described in the Appendix, where the reduced form equation becomes:

$$r = (\frac{1}{b_0 \beta_2 + c_0})(MS^* - b_0 \alpha + b_0 \beta_2 \dot{P}e - U_{md} - b_0 U_{is})$$

Reduced Form of Income

Its reduced form is obtained using the same procedure to obtain that of r:

$$Y = (\frac{1}{b_0 \beta_2 + c_0})(\beta_2 MS^* + c_0 \alpha - \beta_2 c_0 \dot{P}e + c_0 U_{is} - \beta_2 U_{md})$$

However, we will now consider the alternative possibilities with respect to the revision of policy and the estimations of price expectations.

Case A. Price expectations are a function of past income growth.

In this case, price expectations depend upon past income growth rates. Substituting the value of MS* into the reduced form of income above we obtain the reduced form of income under the present assumptions:

$$(Y-Y^*) = \left(\frac{1}{b_0\ \beta_2+c_0}\right)(c_0\beta_2\tilde{\dot{P}}e-c_0\beta_2\dot{P}e+c_0U_{is}-\beta_2U_{md})$$

which shows that deviations of income from target Y* will be affected by differences between estimated and actual price expectations. On the other hand, if $\tilde{\dot{P}}e = \dot{P}e$ then price expectations would not be responsible for income deviations from the target. We now proceed to examine this case under different assumptions, as expressed in Table 3-8 above.

i) MS* is revised every short-term period

i.1) $\dot{P}e$ is perfectly forecasted.

MS* is revised at the begining of each short-term period and $\tilde{\dot{P}}e$ is correctly estimated. Under these circumstances, if $\dot{P}e$ increases (non-randomly), MS* is revised to account for such increase and, since $\dot{P}e$ is correctly estimated, the income target Y* is maintained.

Nothing else occurs since $\dot{P}e$ depends upon income changes and Y* is attained , albeit higher interest rates. Figure 3-16 depicts the analysis in IS-LM terms.

Introducing this assumption into the reduced form of income we obtain:

$$Y_{ij} = Y_{ij}*+(\frac{1}{b_0 \ \beta \ _2+c_0})(c_0 U_{is_{ij}} - \beta \ _2 U_{md_{ij}})$$

i.2) <u>$\dot{P}e$ is not perfectly forecasted.</u>

In this case, the reduced form of income becomes:

$$Y_{ij} = Y_{ij}^{*} + \frac{1}{b_0 \ \beta_2+c_0}(c_0 \ \beta_2(\tilde{\dot{P}}e_{ij}-\dot{P}e_{ij})+c_0 \ U_{is_{ij}} -\beta_2 \ U_{md_{ij}})$$

Where: $(\tilde{\dot{P}}e - \dot{P}e)$ is random.

Figure 3-17 shows the effect on income of such a situation, assuming $\tilde{\dot{P}}e<\dot{P}e$. The initial equilibrium is given by LM_1 and IS_1. If there is a random increase in price expectations, and therefore $\dot{P}e$ are under-estimated by authorities, the actual IS is IS_2 while authorities use $\tilde{IS}_2$ to set the value of the policy instrument. Revision renders a new money supply value according to the estimated IS, which is greater than the money supply needed to bring the economy back to Y^*(point 2 in the figure). The result is that the economy is driven to point 3 in figure 3-17, where income is above target. Furthermore, due to the assumption that price expectations depend upon past income growth, they will grow further as income rises. If this is

Figure 3-16
Model A-4, Case A, Ms* Policy,
Revised in Short-Term Periods: Effect
of a Change in Price Expectations,

Assuming $\tilde{\dot{P}}e = \dot{P}$ e.

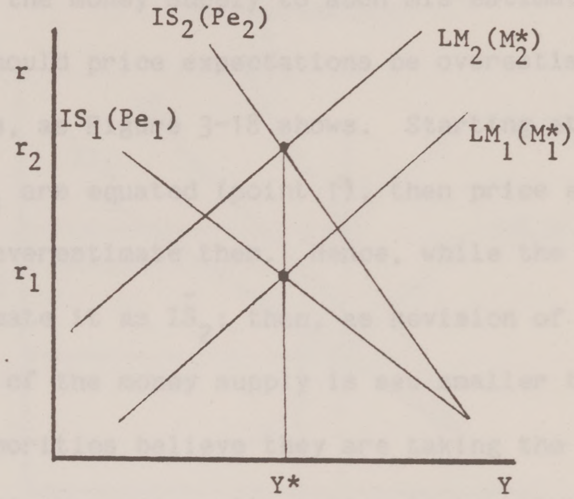
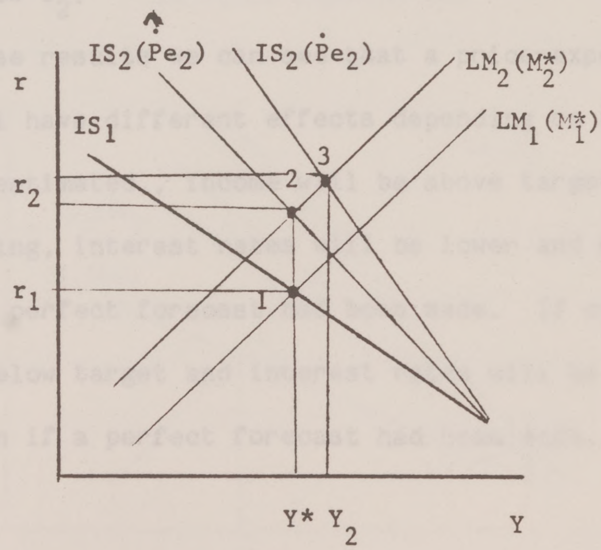


Figure 3-17
Model A-4, Case A, Ms* Policy,
Revised in Short-Term Periods: Effect
of a Change in Price Expectations,

Assuming $\tilde{\dot{P}}e < \dot{P}$ e
(under-estimation)



forecasted by authorities the money supply is adjusted and income remains at Y_2, but if expectations are under-estimated again, income will continue growing and so will interest rates since authorities are trying to adjust the money supply to such mis-estimated price expectations. Should price expectations be overestimated, the situation changes, as Figure 3-18 shows. Starting at the equilibrium where IS_1 and LM_1 are equated (point 1), then price expectations rise but authorities overestimate them. Hence, while the new IS is IS_2, the authorities estimate it as $\tilde{IS}_2$; then, as revision of policy takes place, the value of the money supply is set smaller than the necessary to keep Y^*. Authorities believe they are taking the system to point 2 in the figure, but they are really taking it to point 3, which renders a level of income below target. Furthermore, since income fell, price expectations are adjusted back down and the behavior of income will now depend on how this adjustment is estimated by authorities: if it is correctly estimated, income remains at Y_2, but if it is overestimated it will fall below Y_2.

From these results we can see that a price-expectation mis-forecast will have different effects depending on the nature of the error. If underestimated,, income will be above target and maybe continuously rising, interest rates will be lower and money supply higher than if a perfect forecast had been made. If overestimated,, income will be below target and interest rates will be higher and money supply lower than if a perfect forecast had been made. Now, if the sum

Figure 3-18
Model A-4, Case A, Ms^* Policy,
Revised in Short-Term Periods:
Effect of a Change in Price Expectations

Assuming $\tilde{\dot{P}}e > \dot{P}e$
(overestimation)

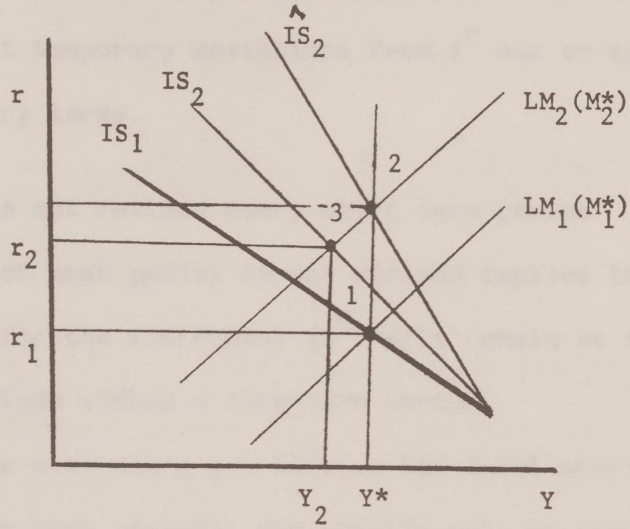
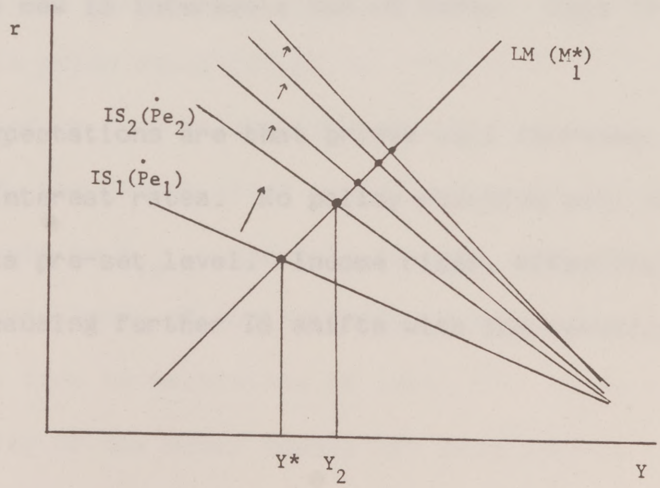


Figure 3-19
Model A-4, Case A, Ms^* Policy, No Revision
in Short-Term Periods: Effect of a Change (positive)
in Price Expectations

$(\dot{P}e > \tilde{\dot{P}}e)$



of such forecasting errors is zero (i.e., $E(\tilde{\dot{P}}e-\dot{P}e) = 0$), then, on average, we can expect that, ceteris paribus, the income target (Y^*) will be achieved. On the other hand, if the correlation between estimation errors in different periods is zero (no autocorrelation), we can expect that temporary deviations from Y^* due to misforecasts of $\dot{P}e$ will not be very large.

ii) Policy is not revised every short term period

The fact that policy is not revised implies that the value initially set for the instrument (M^*) will remain at such level for all short-term periods within a long-term period.

For the case where $j = 1$(i.e., the first short-term period within the long term period), the results will be similar to the revision case above since M^* has just been revised. However, for the cases where $j>1$, changes in price expectations, whether random or due to recent short-run experience, will affect income and interest rates because M^* is not being revised. This means that shifts in the IS due to changes in price expectations will determine the equilibrium of income as the new IS intersects the LM curve. This is shown in figure 3-19.

If expectations are that prices will increase, income will rise and so will interest rates. No policy reaction will occur, since M^* remains at its pre-set level. Income rises, affecting expectations further and causing further IS shifts with the resulting higher income levels.

The reduced form of income is the following:

$$Y_{ij} = Y^*_{ij} + A_{ij} + \frac{1}{b_0\ \beta_2 + c_0}\ (c_o\ \beta_2[-a_{i1}\ (\frac{Y_{i1} - Y_{i-1,4}}{Y_{i-1,4}})$$

$$-a_{i2}\ (\frac{Y_{i2} - Y_{i1}}{Y_{i1}}) - \ldots - a_{ij}\ (\frac{Y_{ij} - Y_{i,j-1}}{Y_{i,j-1}})]$$

Where: $A_{ij} = (c_o\ U_{is_{ij}} - \beta_2\ U_{md_{ij}})(\frac{1}{b_o\beta_2 + c_o})$

Therefore, Y_{ij} is affected by present disturbances on both the real and the monetary sectors (A_{ij}) and by past disturbances as they affect past levels of income.

CASE B. Price Expectations affected by past money growth.

Since the policy procedure under analysis is one in which the money supply is fixed at MS^*, if expectations depend upon MS growth -which itself depends upon MS changes-, expectations will be constant as long as MS^* is maintained. Furthermore, revision of MS^* to adjust for changes in price expectations is irrelevant and income will then not be affected by them unless they change randomly. In the latter case, however, the random shock would be treated as any other random disturbance such as U_{is} . The only further consideration in this case would be an assessment of the validity of the assumption that the money supply can in fact be maintained at level M^*. As we know, the issue of controllability of the money supply has been subject to ample debate.[73]

E. MODEL A-5. Wealth Effect on Consumption.

We now introduce the effect of wealth on consumption, as explained in Chapter 2, resulting in the complete model presented in Table 3-9. Notice that the table shows two alternative cases which differ only in the definition of the Net Worth (NW) variable. We intend to analyze both cases, as economists have different views with respect to the definition of that variable that better represents what people view as wealth.

CASE 1: NW = MS+GD+K.

1. Policy Procedure:Interest Rate Instrument

Determination of Instrument Value(r^*)

Given target income Y^*, the value of the instrument is obtained from the IS equation:

$$r^* = \frac{1}{c_1'+c_1}(Y^*(1-b)-a_1-a_2+T(1+b_1)-G(1+1)-1MS_{t-1}-1GD_{t-1}-1K)$$

73. See, for example, Ralph Bryant, Controlling Money, Brookings Institution, Washington , D.C, 1980.

Table 3-9. Model A-5: Structural Equations and IS-LM Relationships
(wealth effect on consumption included)

Equations*

$$Y = C + I + G$$
$$C = a_1 + b_1\, Yd + c'_1 + 1NW + Uc$$
$$Yd = Y - T$$
$$G = T + \Delta Ms + \Delta GD$$
$$I = a_2 + c_1 r + Ui$$
$$Md = c_0 r + b_0 Y + Umd$$
$$Md = Ms$$
$$NW = GD + MS + K \quad \text{Case 1}$$
$$NW = MS + K \quad \text{Case 2}$$

IS & LM**

$$\text{LM:} \quad Y = \frac{1}{b_0}[Ms - c_0 r - Umd]$$

$$\text{IS=} \quad Y = \frac{1}{1-b_1}\Big[a_1 + a_2 - T(b_1+1) + 1Gd_{t-1} + 1MS_{t-1}$$

$$\text{Case 1} \qquad + G(1+1) \Big] + r\,\frac{[(c'_1 + c_1)]}{1-b_1} + \frac{1}{1-b_1}\,Ui + Uc$$

$$\text{Case 2} \qquad Y = \frac{1}{1-b_1}[a_1 + a_2 + (G - b_1 T) + 1K] + \frac{1}{1-b}MS + \frac{(c'_1 + c_1)r}{1-b_1} + \frac{Ui + Uc}{1-b_1}$$

---

where: Y = income; c = consumption; I = investment; G = government expenditure; T = taxes; Yd = disposable income; r = interest rate; NW = net worth; GD = government debt; Ms = money supply; K = stock of capital
Uc, Ui, Umd = disturbance terms. Δ = difference operator;
It is assumed that: $C:<0$, $c_1<0$, $co<0$, $a_1, a_2>0$
$0<1<1$, $0<b_0<1$, $0<b_1<1$.

---

* Source: Chapter 2 above and Yiannis P. Venieris and Frederick D. Sebold, Macroeconomic Models and Policy, Wiley and Sons, pp. 489-503.

** See Appendix, Chapter 3 derivations.

Implications of the Procedure

Under the present assumptions, the slope of the IS has changed to:

$$\beta_1 = \frac{c_1 + c_1'}{1-b_1}$$

while in the previous models it was $\beta = \frac{c_1}{1-b_1}$. Clearly, $|\beta_1| > |\beta|$.

Furthermore, we have new variables affecting the IS which are predetermined or exogenous from both the real and the monetary sectors. A comparison between the IS before and after the Wealth effect is done in Figure 3-20.

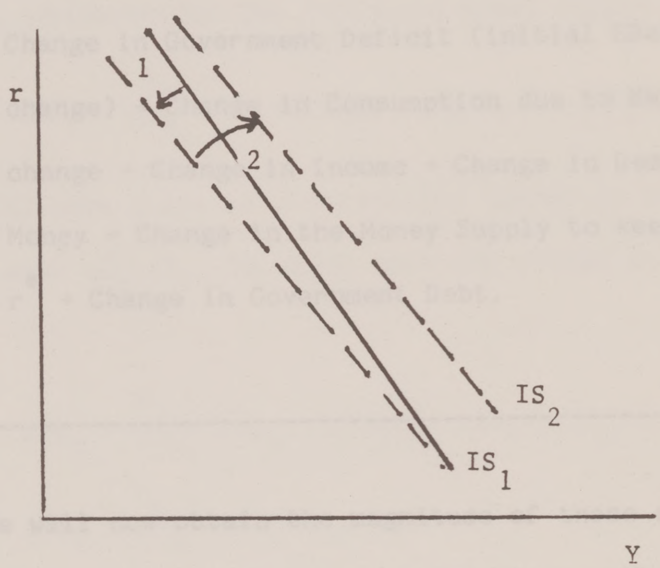
If interest rates are used as instruments, the money supply becomes endogenous and so will government debt and net worth (NW). To identify the different factors that will affect the money supply, we look at the money market equilibrium and obtain the reduced form for the money supply:

$$MS = C_o r^* + b_o \left(\frac{a_1 + a_2}{1-b_1} + \frac{1}{1-b_1}(G_t - T_t) + \frac{G_t - b_1 T_t}{1-b_1} + \frac{1}{1-b_1}\Sigma(G_{t-i} - T_{t-i}) + \frac{1}{1-b_1} GD_{t-n-1} \frac{1}{1-b_1} MS_{t-n-1} + r^* \frac{c_1' + c_1}{1-b_1} + \frac{U_1 + U_c}{1-b_1}\right) + U_{md}$$

Where: $0<1<1$, $0<b_1<1$, $0<b_o<1$, $c_1' <0$ and $c_1 <0$.

The present framework gives us the opportunity to analyze fiscal policy in the context of monetary policy , as will be clear next.

Figure 3-20
Model A-5: Effect of the Introduction
of the Wealth Effect in the IS



IS_1: Before inclusion of wealth effect on consumption.

IS_2: Including wealth effect on consumption.

Under an interest rate policy instrument, we can see that the reduced form for the money supply shows the fact that increasing government deficits are financed at least in part by expanding the money supply; the expansion of the money supply will depend upon the effect of such government deficit in the demand for money. The complete transmission mechanism is the following:

> Change in Government Deficit (initial GDebt change) → Change in Consumption due to NW change → Change in Income → Change in Demand for Money → Change in the Money Supply to keep r^* → Change in Government Debt.

We will now obtain the magnitude of these effects on the money supply, using the reduced form equation above. When both government expenditures and taxes change, the money supply will be altered as follows:

$$\Delta MS_t = \frac{b_o l}{1-b_1}(\Delta G_t - \Delta T_t) + b_o \frac{\Delta G_t - b_1 \Delta T_t}{1-b_1}$$

which comes from:

$$\Delta MS_t = b_o \Delta Y_t = \Delta Md.$$

Where: $\Delta Y_t = \frac{l}{1-b_1}(\Delta G_t - b_1 \Delta T_t) + \frac{\Delta G_t - b_1 \Delta T_t}{1-b_1}.$

From this we can distinguish two possibilities: (1) When there is not an increase or decrease in the government deficit; this means, $\Delta G_t = \Delta T_t$, which causes the following money supply change:

$$\Delta MS_t = \frac{b_o}{1-b_1}(\Delta G_t - b_1 \Delta G_t) = b_o \Delta G_t = b_o \Delta T_t.$$

(2) When there is a deficitary change in the government budget; that is, when $\Delta G_t > \Delta T_t$, in which case the change in the money supply becomes:

$$\Delta MS_t = \frac{b_o l}{1-b_1}(\Delta G_t - \Delta T_t) + \frac{b_o}{1-b_1}(\Delta G_t - b_1 \Delta T_t).$$

or $$\Delta MS_t = \frac{b_o l}{1-b_1}(\Delta G_t - \Delta T_t) + b_o \gamma$$

where $\gamma > \Delta G_t$, $\gamma = \frac{\Delta G_t - b_1 \Delta T_t}{1-b_1} > \Delta G_t$.[74]

---

[74] If $\Delta G = \Delta T$, $\gamma = \frac{\Delta G - b_1 \Delta G}{1-b_1} = \Delta G$. Hence if $\Delta G > \Delta T$, $\gamma = \frac{\Delta G - b_1 \Delta T}{1-b_1} > \Delta G$.

Both of these possibilities are consistent with either a balanced or an unbalanced budget, as Table 3-10 indicates. In such Table we give the effects of the budgetary changes on the money supply of period t and of period t+1, since, as the reduced form equation for MS shows, MS_t is affected by $G_{t-1}-T_{t-1}$ through the term:

$$\frac{b_o 1}{1-b_1} \Sigma(G_{t-i}-T_{t-i}).$$

Such effect ends out being equal to:

$$\Delta MS_t = \frac{b_o 1}{1-b_1}(G_{t-1}-T_{t-1}).$$ [75]

On the other hand, the reduced form equation for the Government Debt shows the part of the increasing government deficits that come to be financed by government debt in the new equilibrium:

$$GD_t = (G_t-T_t)+(1-\frac{1\ b_o}{1-b_1})(GD_{t-n-1}+MS_{t-n-1}+\Sigma(G_{t-i}-T_{t-i}))$$

$$-c_o r^* - \frac{b_o}{1-b_1}(a_1+a_2) - \frac{b_o 1(G_t-T_t)}{1-b_1} - \frac{b_o(G_t-b_1T_t)}{1-b_1}$$

$$-r^*(c'_1+c_1)\frac{b_o}{1-b_1}-(U_i+U_c)\frac{b_o}{1-b_1}-U_{md}.$$

Then, when there is a change in the government budget variables Δ G and ΔT, government debt is affected as follows:

---

75. See Table 3-10. For proof, see Appendix.

Table 3-10. Model A-5; Case 1; Interest Rate Policy (r^*): Effects of a Change in Government Budget on Money Supply and Government Debt

Budget changes period t	Resulting Budget position period t	Effects on MS_t	Effects on MS_{t+1}	Effects on GD_t
$\Delta G=\Delta T$	A. $G_{t-1}=T_{t-1}$ Balanced hence $G_t=T_t$ Budget	$\Delta MS_t=b_0\Delta G_t=b_0\Delta T_t$	0	$\Delta GD_t=-b_0\Delta G_t = -b_0\Delta T$
	B. $G_{t-1} > T_{t-1}$ Same hence $G_t > T_t$ Deficit	$\Delta MS_t=b_0\Delta G_t=b_0\Delta T_t$	$\Delta MS_{t+1} = \frac{b_0 l}{1-b_1}(G_t-T_t)$	$\Delta GD_t=-b_0\Delta G_t$
	C. $G_{t-1} < T_{t-1}$ Same Superavit	$\Delta MS_t=b_0\Delta G_t=b_0\Delta T_t$	$\Delta MS_{t+1} = \frac{b_0 l}{1-b_1}(T_t-G_t)$	$\Delta GD_t=-b_0\Delta G_t$
$\Delta G > \Delta T$	A. $G_{t-1}>T_{t-1}$ increased $G_t>T_t$ Deficit	$\Delta MS_t=b_0\ Y + \frac{b_0 l}{1-b_1}(\Delta G-\Delta T)$ where $Y = \frac{(\Delta G-b_1\Delta T)}{1-b_1}$	$\Delta MS_{t+1} = \frac{b_0 l}{1-b_1}(G_t-T_t)$	$\Delta GD_t = -b_0\ Y + \left(1-\frac{lb_0}{1-b_1}\right)(\Delta G-\Delta T)$
	B. $G_{t-1} = T_{t-1}$ new $G_t>T_t$ Deficit	same	same	same
	C. $G_{t-1}<T_{t-1}$ Decreased Superavit	same	$\Delta MS_{t+1} = \frac{b_0 l}{1-b_1}(G_t-T_t) \lessgtr 0$	same

Table 3-10 (continued)

Budget changes Period t	Effects on GD_{t+1}	Effects on Y_t	Effectson Y_{t+1}
$\Delta G = \Delta T$	0	A. $\Delta Y_t = \frac{1}{1-b_1}(\Delta G - b_1 \Delta T)$ $\Delta Y_t = \Delta G_t$	0
	$\Delta GD_{t+1} = \left(1 - \frac{lb_{\ell}}{1-b_1}\right)(G_t - T_t)$	B. $\Delta Y_t = \Delta G_t$	$\Delta Y_{t+1} = \frac{1}{1-b_1}(G_t - T_t)$
	$\Delta GD_{t+1} = \left(1 - \frac{lb_{\ell}}{1-b_1}\right)(G_t - T_t)$	C. $\Delta Y_t = \Delta G_t$	$\Delta Y_{t+1} = \frac{1}{1-b_1}(G_t - T_t) < 0$
$\Delta G > \Delta T$	$\Delta GD_{t+1} = \left(1 - \frac{lb_{\ell}}{1-b_1}\right)(G_t - T_t)$	$\Delta Y_t = \frac{1}{1-b_1}[l(\Delta G - \Delta T) + (\Delta G - b_1 \Delta T)]$	$\Delta Y = \frac{1}{1-b_1}(G_t - T_t)$
$\Delta G > \Delta T$	$\Delta GD_{t+1} = \left(1 - \frac{lb_{\ell}}{1-b_1}\right)(G_t - T_t)$	$\Delta Y_t = \frac{1}{1-b_1}[l(\Delta G - \Delta T) + (\Delta G - b_1 \Delta T)]$	$\Delta Y = \frac{1}{1-b_1}(G_t - T_t)$
$\Delta G > \Delta T$	$\Delta GD_{t+1} = \left(1 - \frac{lb_{\ell}}{1-b_1}\right)(G_t - T_t) \lessgtr 0$	$\Delta Y_t = \frac{1}{1-b_1}[l(\Delta G - \Delta T) + (\Delta G - b_1 \Delta T)]$	$\Delta Y = \frac{1}{1-b_1}(G_t - T_t) \lessgtr 0$

where:

G_{t-i} = government expenditure period t-i.

T_{t-i} = tax revenues period t-i.

MS_t = money supply period t.

GD_t = government debt period t.

For b_0, l, b_1 coeficients, see table 3-9.

$$\Delta GD_t = \Delta G_t - \Delta T_t - \frac{lb_o}{1-b_1}(\Delta G_t - \Delta T_t) - \frac{b_o}{1-b_1}(\Delta G_t - b_1 \Delta T_t).$$

From there, if the changes in G and T are equal, we have that:

$$\Delta GD_t = -b_o \Delta G_t.$$

But if the change in G is greater than the change in T, we have:

$$\Delta GD_t = (1 - \frac{lb_o}{1-b_1})(\Delta G_t - \Delta T_t) - b_o \gamma$$

Where: $\gamma = \dfrac{\Delta G_t - b_1 \Delta T_t}{1-b_1} > \Delta G_t.$

The reduced form for GD_t shows that the ΔGD_t equals the increase in the deficit minus the amount of such deficit that generated money supply expansion, due to the policy of keeping r^*. Furthermore, the equation shows that GD_{t+1} is also affected by the deficit of period t:

$$\Delta GD_{t+1} = (1 - \frac{lb_o}{1-b_1})(G_t - T_t).$$[76]

Recall that $\Delta MS_{t+1} = \frac{lb_o}{1-b_1}(G_t - T_t)$; therefore, GD_{t+1} is also the residual from the effect of the deficit $(G_t - T_t)$ on MS_{t+1}.

The implications of the above results are analyzed with more detail in the following section, together with the effects on income.

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76. See Appendix for proof.

Reduced Form of Income

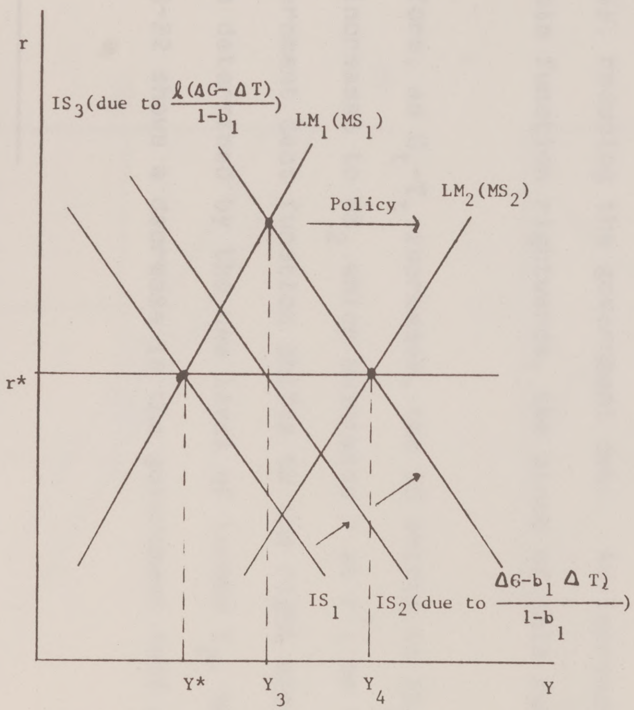
The reduced form is given by:

$$Y = \frac{1}{1-b_1}[a_1+a_2+l(G_t-T_t)+(G_t-b_1T_t)+l\Sigma(G_{t-i}-T_{t-i})$$
$$+lGD_{t-n-1}+lMS_{t-n-1}+r^*(c_1'+c_1)+U_i+U_c]$$

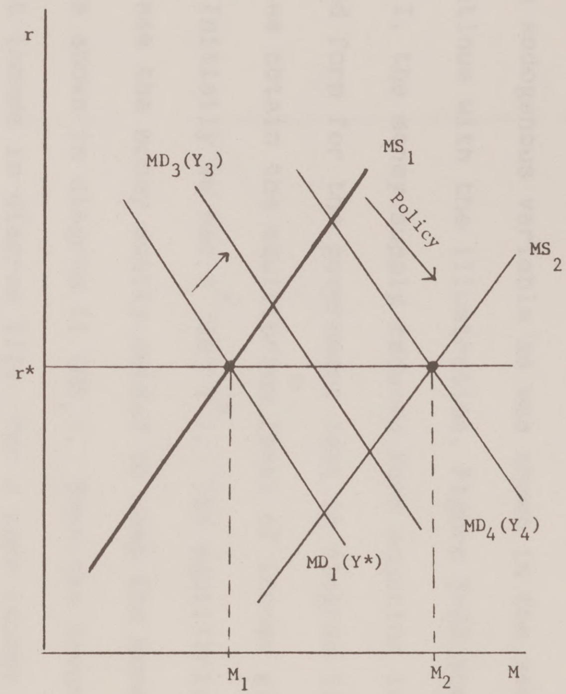
The equation shows that only IS disturbances would cause income to change; furthermore, it reveals that current and past increases in government deficits, as well as deficits perse, will also affect present income.

We now proceed to illustrate the trasmission mechanism of a change in the Government deficit, under the present assumptions. Suppose that $\Delta(G_t-T_t)>0$; the effects start to operate in the goods market as follows: there is an additional demand for goods and services that affects income, thus shifting the IS rightwards. Before the money supply increases, the deficit is financed with government debt (GD_t increases) and therefore, net worth (NW) increases as well as consumption. The IS shifts further right. In Figure 3-21 as the demand for money increases due to higher income, interest rates start rising above the target level r^* and therefore monetary policy is called into action: the money supply will be increased to bring r back to r^*. (See Diagram II). As the money supply expands to Ms_2, the LM moves right until r^* is achieved, given IS_3. The new equilibrium is set at r^*, Y_4, LM_2 and M_2. Therefore, the money supply has expanded to

Figure 3-21
Model A-5. Interest Rate Policy:
Effect of a Change in G and T.



I. IS-LM equilibrium



II. Money market equilibrium

attain the target level of interest rates, but income is off-target. The effects on government debt need yet to be analyzed, since this has also become an endogenous variable as was shown in the section above.

To continue with the illustration, Figure 3-22 shows the IS and LM in diagram I, the money supply reduced form equation in diagram II, and the reduced form for the government debt in diagram III. Starting at diagram I, we obtain the equilibrium level of income given the IS and the r^*. (Initially we had r^* and Y^*). The equilibrium level of income determines the money supply needed to keep the money market in equilibrium, as shown in diagram II (MS_1). Then the Government debt is plotted against income in diagram III: for a zero income level government debt equals the deficit (G-T), but as income rises, money supply rises and hence part of the deficit is financed by an increase in money supply, reducing the government debt. An increase in (G-T) would shift this function rightwards, the slope of this function being $-b_o$.[77]

Therefore, as G_t-T_t increases, the IS shifts to IS_2, and the money supply increases to MS_2 which maintains r at r^*; on the other hand, the government debt function shifts to the right and the new level of GD is determined by the new level of income Y_2, hence giving GD_2. Figure 3-22 shows a decrease in the government debt as the result

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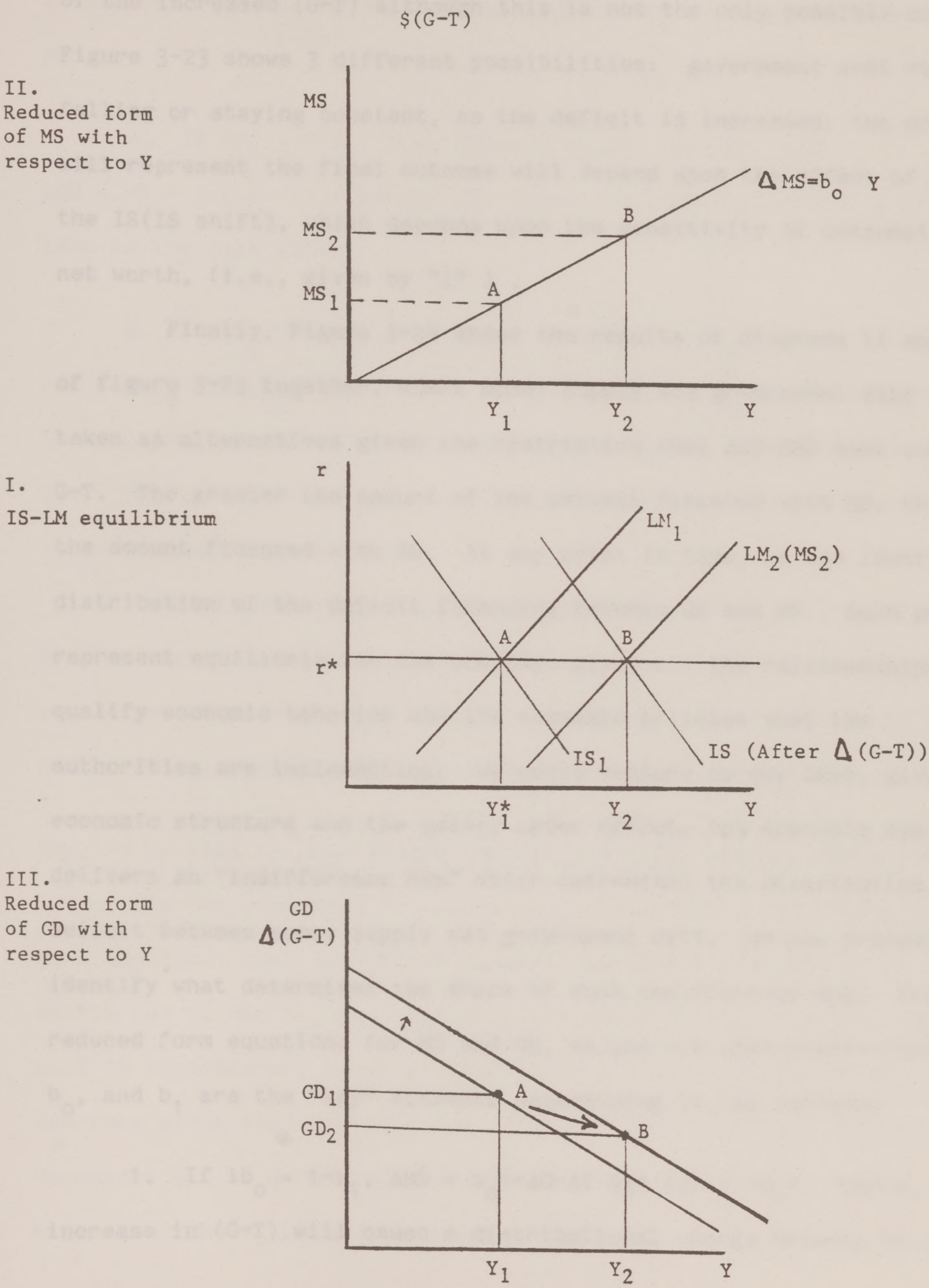
77. Recall that $\Delta GD_t = G_t-T_t-\Delta MS_t$, which is tantamount to: $\Delta GD_t = G_t-T_t-b_o\Delta Y$

Figure 3-22
Model A-5, Case 1 Interest Rate Policy:
Effect of a Change in the Budget



of the increased (G-T) although this is not the only possible outcome. Figure 3-23 shows 3 different possibilities: government debt rising, falling or staying constant, as the deficit is increased; the one that will represent the final outcome will depend upon the effect of G-T in the IS(IS shift), which depends upon the sensitivity of consumption to net worth, (i.e., given by "l") .

Finally, Figure 3-24 shows the results of diagrams II and III of figure 3-23 together, where money supply and government debt are taken as alternatives given the restriction that $\Delta GD+\Delta MS$ must equal G-T. The greater the amount of the deficit financed with MS, the lower the amount financed with GD. At any point in time, we can identify the distribution of the deficit financing between GD and MS. Such points represent equilibria for the economy, given all the relationships that qualify economic behavior and the economic policies that the authorities are implementing. We could venture to say that, given the economic structure and the policy under effect, the economic system delivers an "Indifference Map" which determines the distribution of the deficit between money supply and government debt. We now proceed to identify what determines the shape of such indifference map. From the reduced form equations for MS and GD, we can see that coefficients l, b_o, and b_1 are the "key" elements determining it, as follows:

1. If $lb_o = 1-b_1$, $\Delta MS = b_oY+\Delta G-\Delta T$ and $\Delta GD = -b_oY$. Hence, an increase in (G-T) will cause a distributional change between MS and GD

as Diagram I in figure 3-24 shows, biased towards a larger share for MS.

2. If $lb_o > 1-b_1$, then $\Delta MS = b_o Y + \frac{lb_o}{1-b_1}(\Delta G - \Delta T)$, and $\Delta GD = -b_o Y - \left| \frac{lb_o}{1-b_1} - 1 \right| (\Delta G - \Delta T) < 0$. Here, the decrease in GD is greater than in the case above.

3. If $lb_o < 1-b_1$, then $\Delta MS = b_o Y + \frac{lb_o}{1-b_1}(\Delta G - \Delta T)$, and $\Delta GD = -b_o Y + (1 - \frac{lb_o}{1-b_1})(\Delta G - \Delta T)$. In this case, if $(1 - \frac{lb_o}{1-b_1}(\Delta\ G - \Delta T) > b_o Y$, then GD will increase as diagram II in Figure 3-24 shows (from point 1 to 3). This latter situation would be one in which the indifference map is not biased towards the MS.

A conclusion here is that the lower b_o, l and b_1 are, the less biased towards MS will the indifference curves be, which implies that a change in (G-T) will bring a smaller or no increase in the proportion of the deficit that is financed with money supply expansion.

Now, we go back to the reduced form equation for income, and proceed to illustrate the mechanics of a monetary disturbance , which according to such equation, does not affect income. As figure 3-25 shows, if the LM shifts due to the $U_{md} > 0$, interest rates start rising, monetary policy will be expansionary in order to restore r^*, and government debt will decrease given that G-T is fixed. The net worth variable remains unchanged and as a result Income Y^* is restored. Hence, although the model is dynamic, given the fixity of G-T the effect of changes in the

Figure 3-23
Model A-5, Case 1, Interest Rate Policy:
Different Possible Effects of a Change
In (G-T) on Government Debt.

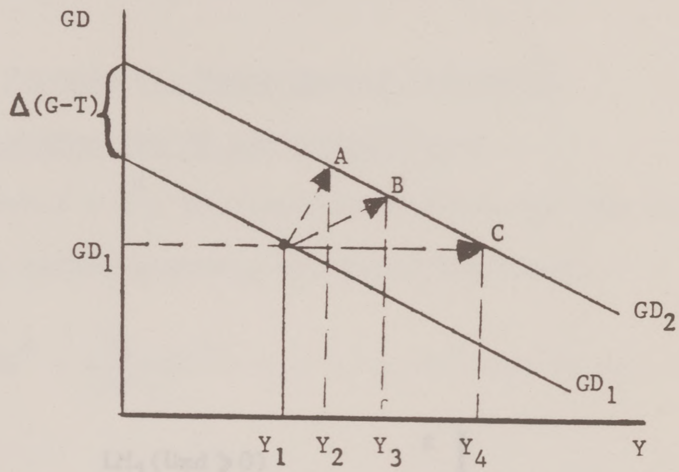


Figure 3-24
Model A=5, Case 1, Interest Rate Policy:
Money Supply Government Debt Trade Off
and the Implicit Social Indifference Map

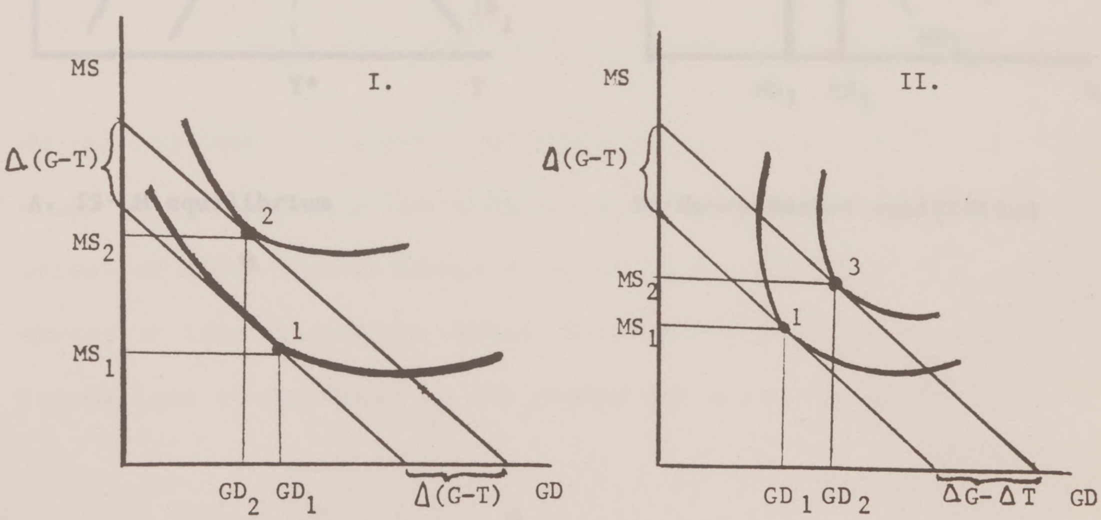
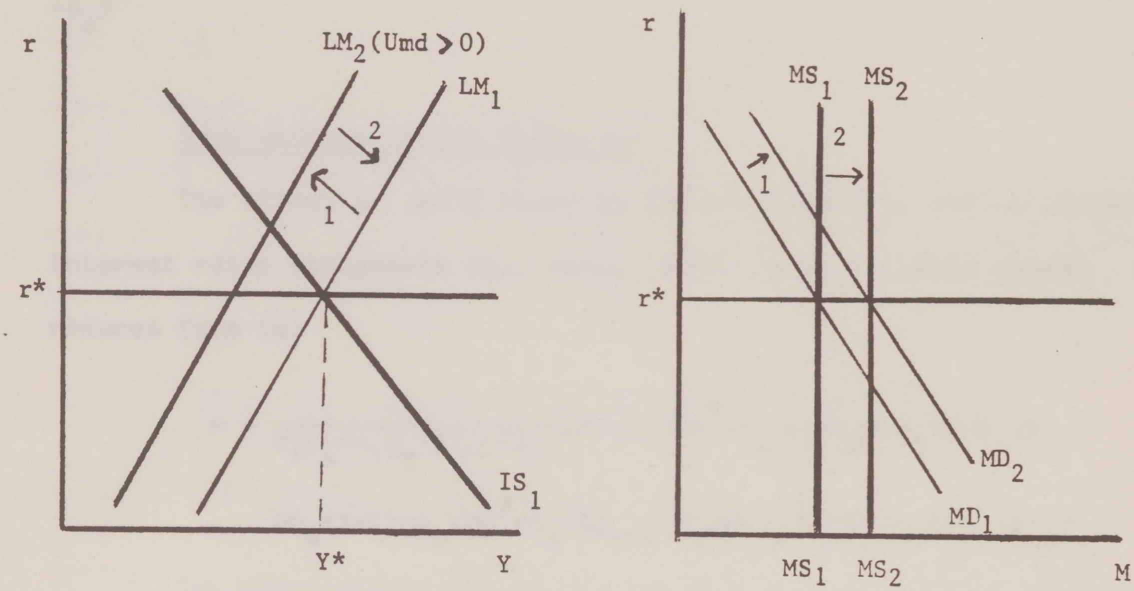


Figure 3-25
Model A-5. Effect of a Demand for Money Shock
On Income Under an Interest Rate Policy
Case A(NW + MS + GD + K)



A. IS-LM equilibrium

B. Money market equilibrium

money supply is offset by opposite changes in government debt. In other words,

$$\Delta MS_t = -\Delta GD_t, \text{ as long as G-T is fixed.}$$

2. Policy Procedure: Money Supply Instrument

Determination of Instrument Value

Given $Y = Y^*$, the instrument value for the money supply is obtained as before yielding the following result:

$$MS^* = \frac{c_o}{c_1' + c_1}[Y^*(1-b_1)-a_1-a_2+T(1+b_1)-G(1+L) -lMS_{t-1} -lGD_{t-1} -lK] + b_oY^*$$

Implications of the Procedure

The effect of using money as the instrument is that it makes interest rates endogenous and, hence, affected by economic shocks. The reduced form is:

$$r = \frac{1}{c_1'b_o+c_1b_o+c_o(1-b_1)}[(1-b_1)MS^*-b_o(a_1+a_2)+b_o(1+b_1)T- Gb_o(1+l)-b_olMS^*-b_olGD_{t-1}-b_olK-b_o(U_c+U_i)-(1-b_1)U_{md}]$$

We can see that the higher the income elasticity of the demand for money (b_o) relative to the marginal propensity to save, the lower the effect of money demand shocks relative to the effect of real sector shocks on interest rates. Hence, in an economy in which increases in income lead to increases in the demand for money ($b_o\Delta Y$) which are proportionally greater than the increases in savings($(1-b_1)\Delta Y$), and in which the money supply is used as the monetary policy instrument, shocks in the real sector will cause greater interest rate changes than monetary shocks, given a money supply instrument.

On the other hand, under the present policy procedure, since the money supply is held constant (MS^*) then $\Delta MS = 0$, with the implication that government debt becomes exogenous, depending only upon changes in the deficit. This result is obtained from:

$$G = T+\Delta MS+\Delta GD$$

Then if $\Delta MS = 0$, $G-T = \Delta GD$, which implies that government deficits are solely financed by government debt expansions. Let us now obtain the magnitude of the effects of government deficits on interest rates as well as in government debt. Assuming a change in both G and T, the effects are the following:

$$\Delta r = \frac{1}{c'_1 b_o+c_1 b_o+c_o(1-b_1)}(b_o(1+b_1)\Delta T-b_o(1+1)\Delta G).$$

Where $\quad \nu\# = \dfrac{1}{c'_1 b_o+c_1 b_o+c_o(1-b_1)} < 0.$

The above expression for Δr can also be expressed as follows:

$$\Delta r = \nu(-b_o(\Delta G-b_1\Delta T)-b_o 1(\Delta G-\Delta T)).$$

We can also see that the interest rate reduced form includes GD_{t-1}, which is closely related to government deficits. Let us recall that:

$$GD_{t-1} = GD_{t-n-1} + MS_{t-n-1} - MS_{t-1} + \Sigma(G_{t-i} - T_{t-i})$$

Hence: $\Delta GD_{t-1} = \Delta MS_{t-1} + \Delta(\Sigma(G_{t-i} - T_{t-i}))$

since GD_{t-n-1} and MS_{t-n-1} are the GD and MS levels for some past initial period and are thus constant. Then, since under a money supply instrument procedure $\Delta MS_{t-1} = 0$, we have:

$$\Delta GD_{t-1} = G_{t-1} - T_{t-1}$$

Since $\Delta r_t = \nu(-b_o l \Delta GD_{t-1})$, then:

$$\Delta r_t = \nu(-b_o l(G_{t-1} - T_{t-1}) + \nu(-b_o(\Delta G_t - b_1 \Delta T_t) - b_o l(\Delta G_t - \Delta T_t)$$

which indicates that a deficit in the present will affect future interest rates and not only current ones.

Reduced Form of Income

As the following formula shows, both IS and LM disturbances affect income.

$$Y = \frac{1}{b_o(c_1' + c_1) + c_o(1-b_1)}[(c_1' + c_1 + c_o l)MS^* + c_o(a_1 + a_2) - c_o(1+b_1)T$$

$$+ c_o(1+l)G + c_o l GD_{t-1} + c_o lK + c_o U_i + c_o U_c - (c_1' + c_1)U_{md}].$$

From this equation, it is shown that the greater the interest sensitivity of the demand for money (and hence the greater the LM slope $\frac{\Delta Y}{\Delta r}$, the greater the effect of real sector's shocks on income. On the other hand, the larger the interest elasticity of Investment and consumption (c_1 and c_1' respectively), the larger the effect of money demand shocks on income.[78]

Finally, the effects of an increase in the deficit are given by:

$$\Delta Y_t = \nu(-c_o(1+b_1)\Delta T_t+c_o(1+1)\Delta G_t)$$

or: $$\Delta Y = \nu(c_o(\Delta G_t-b_1\Delta T_t)+c_o 1(\Delta G_t-\Delta T_t))$$

where $\nu<0$.

The effect of the deficit on income depends upon the coefficients c_o and 1, plus those included in ν. Hence, the interest sensitivity of the demand for money plays a very important role.

The effects of a deficit on future income changes are given by:

$$\Delta Y_{t+1} = \nu(c_o\ 1\ )\Delta GD_t$$

where: $\Delta GD_t = G_t-T_t$.

Table 3-11 summarizes the effects of a deficit increase under the MS^* policy, on interest rates, income and government deficit.

CASE 2: NW = MS+K

1. Policy Procedure :Interest Rate Instrument

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78. See Appendix for a proof of both statements.

Table 3-11. Model A-5; Case 1; Money Supply Instrument: Effects of a Change in G and T on Interest Rate, Income and Government Debt.

Budget changes period t	Resulting Budget position period t	Effects on r_t	Effects on r_{t+1}	Effects on income $(Y)_t$
$\Delta G = \Delta T$	A. $G_{t-1} = T_{t-1}$ Balanced hence $G_t = T_t$ Budget	$\Delta r_t = b_0 \nu(-b_0(\Delta G - b_1 \Delta T)$ where $\nu = \frac{1}{b_0(c'_1 + c_1) + c_0(1-b_1)}$	0	$\Delta Y_t = \nu(c_0(\Delta G_t - b_1 \Delta T_t))$
	B. $G_{t-1} > T_{t-1}$ Same hence $G_t > T_t$ Deficit	same	$\Delta r_{t+1} = \nu(-b_0 l(G_t - T_t))$	same
	C. $G_{t-1} < T_{t-1}$ same	same	same	same
$\Delta G > \Delta T$	A. $G_{t-1} > T_{t-1}$ increased $G_t > T_t$ Deficit	$\Delta r_t = \nu \left[b_0(\Delta G - b_1 \Delta T) - b_0 l(\Delta G - \Delta) \right]$	same	$\Delta Y_t = \nu \left[c_0(\Delta G_t - b_1 \Delta T_t) + c_0 l(\Delta G_t - \Delta T_t) \right]$
	B. $G_{t-1} = T_{t-1}$ new $G_t > T_t$ Deficit	same	same	same
	C. $G_{t-1} < T_{t-1}$ decreased	same	same $\lessgtr 0$	same

Table 3-11 (continued)

Budget changes Period t	Effects on Y_{t+1}	Effects on GD_t (exogenous)	Effects on GD_{t+1} (exogenous)
$\Delta G = \Delta T$	0	$\Delta GD_t = \Delta G - \Delta T = 0$	0
	$\Delta Y_{t+1} = \nu(c_o 1)(G_t - T_t)$	0	0
	same	0	0
$\Delta G > \Delta T$	same	$\Delta GD_t = \Delta G - \Delta T$	0
	same	same	0
	same $\lesseqgtr 0$	same	0

Source: Reduced form equations for ν_t and Y_t. See text, model A-5, Case 1, Money Supply Instrument.

Determination of the instrument value

Under the assumption that net worth is defined as the sum of money supply and capital stock only, the first consequence is a different IS than that in Case 1:[79]

$$Y = \frac{1}{1-b}(a_1+a_2+(G-b_1T)+lK+lMS+r(c_1'+c_1)+U_i+U_c)$$

As the equation shows, the money supply appears in the IS implying that the value of the instrument r^* cannot be determined, as before, solely by the IS for a given Y^*. It is now necessary to include the monetary sector.[80] The resulting formula to determine the value of r^* is:

$$r^* = \frac{1}{c_1'+c_1+lc_o}[Y^*(1-b_1-lb_o)-a_1-a_2+b_1T-G-lK]$$

Implications of the Procedure

As before, the interest rate policy procedure implies that the money supply becomes endogenous. Such endogeneity of MS has different consequences than in previous cases and models since now it is also part of the real sector(i.e., IS). Its reduced form is the following:

$$MS = \frac{1}{1-b_1-b_ol}[(c_o-c_ob_1+b_oc_1'+b_oc_1)r^*+b_o(a_1+a_2+G-b_1T+lK)$$
$$+b_o(U_i+U_c)+(1-b_1)U_{md}]$$

---

79. For derivations see Appendix.

80. For the new derivations see Appendix

where we assume $(b_o l)<(1-b_1)$ for a stable system.

In the formula, the coefficient $\frac{1}{1-b_1-b_o l}$ accounts for the effect of money supply increases on consumption and back on money supply due to the policy of keeping r at r^*.

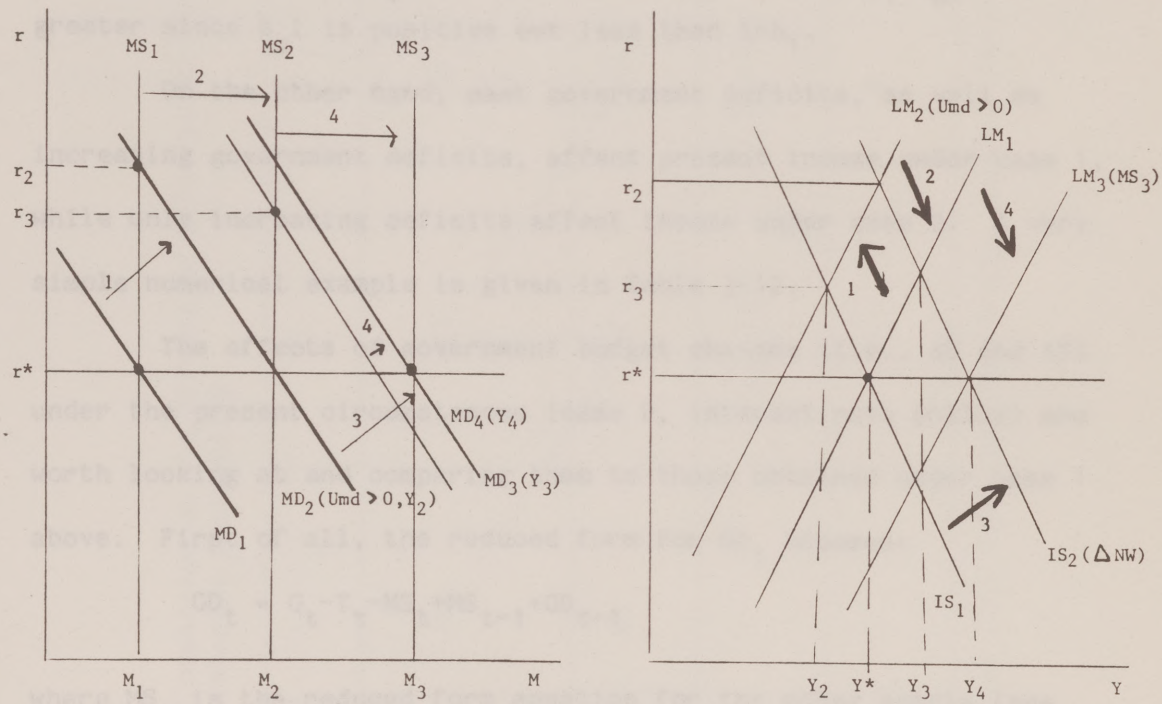
Reduced form of Income

The reduced form equation is:

$$Y = \frac{1}{1-b_1-b_o l}[a_1+a_2+G-b_1T+lK+(c_1'+c_1+lc_o)r^* \quad +U_c+U_i+lU_{md}]$$

As this equation shows, even under an interest rate policy monetary sector shocks will cause income to deviate from target. A comparative statics illustration seems adequate at this point in order to capture the elements involved under the present analysis. Suppose $U_{md}>0$(money demand shock); both the demand for money and the LM shift, as shown in figure 3-26, to MD_2 and LM_2, respectively . Since the interest rate rises above r^*, the money supply increases to restore it, and hence income Y^* is restored(LM goes back to LM_1 in the figure). However, since the money supply has increased, net worth has increased and so has consumption, causing the IS to shift rightwards to IS_2. The result is a higher income that further causes money demand to expand, causing interest rates to rise, too. Money supply expansion will again result in bringing interest rates back to r^*. Thus, M_2 moves to M_3 and LM_1 to LM_3 in the figure, leading to a level of income above target Y^*. Since the money supply has risen again, consumption will rise too, and

Figure 3-26
Model A-5. Effects of an LM Shock under
an Interest Rate Policy
Case 2 (NW + MS + K).



this process will continue until a new equilibrium is attained.[81]

Finally, it is interesting to compare cases 1 and 2 under the interest rate policy, given that the only difference between them is the definition of net worth.

In case 1, LM shocks do not affect equilibrium income, while they do in case 2. In addition, in case 2 the multiplier of IS shocks is greater than that of case 1. While in case 1 such effect is given by the coefficient $\frac{1}{1-b_1}$, in case 2 it is given by $\frac{1}{1-b_1-b_o l}$ which is greater since $b_o l$ is positive but less than $1-b_1$.

On the other hand, past government deficits, as well as increasing government deficits, affect present income under case 1, while only increasing deficits affect income under case 2. A very simple numerical example is given in Table 3-12.

The effects of government budget changes (i.e., ΔG and ΔT) under the present circumstances (case 2, interest rate policy) are worth looking at and comparing them to those obtained under case 1 above. First of all, the reduced form for GD_t becomes:

$$GD_t = G_t - T_t - MS_t + MS_{t-1} + GD_{t-1}$$

where MS_t is the reduced form equation for the money supply (see above). Hence, all factors affecting the money supply will affect

81. Equilibrium is attained under the assumption that $lb_o<(1-b_1)$, as is shown in the appendix to this chapter.

Table 3-12. Comparison of the Effects of Deficits on Income under Cases 1 and 2, Assuming an Interest Rate Policy

$t =$	1	2	3	4
$\sum_{i=1}^{t} (G-T)_i$	100	120	140	160
$(G-T)_t$	20	20	20	20
ΔY Case 1	--	$\frac{1}{1-b_1}(20)$	$\frac{1}{1-b_1}(20)$	$\frac{1}{1-b_1}(20)$
ΔY Case 2	--	0	0	0

Source: Reduced form equations for income, Cases A and B.

government debt in the opposite direction, as long as G_t, T_t, MS_{t-1} and GD_{t-1} remain constant.

Table 3-13 shows the effects of changes in G and T on the key endogenous variables MS, Y and GD. In this case (case 2), an increase in the deficit does not affect the net worth until the money supply has been increased as a result of the deficit increase; in contrast, in case 1 the deficit affects net worth immediately through a change in government debt, which in such case is assumed to be included in the public's concept of wealth. The trasmission mechanisms are the following:

<u>Case 2:</u>

(G-T) -----multiplier_2 → ΔY → ΔMd

Δconsumption ← Δwealth ← ΔMS to keep r^* ←

<u>Case 1:</u>

$\Delta(G-T)$ → multiplier_1 → ΔY

ΔGovernment debt ← ΔMS ← ΔMd

ΔWealth(net worth) → ΔConsumption →

Table 3-13. Model A-5, Case 2, Interest Rate Policy: Effects of a Change in G and T on Money Supply, Income and Government Debt

Budget changes	Effects on MS_t	Effect on Y_t	Effect on GD_t
$\Delta G_t = \Delta T_t$	$\Delta MS_t = \frac{b_0(1-b_1)\Delta G}{1-b_1-b_0 l}$	$\Delta Y_t = \frac{\Delta G(1-b_1)}{1-b_1-b_0 l}$	$\Delta GD_t = \frac{-b_0(1-b_1)\Delta G}{1-b_1-b_0 l}$
$\Delta G_t > \Delta T_t$	$\Delta MS_t = \frac{b_0(\Delta G-b_1\Delta T)}{1-b_1-b_0 l}$	$\Delta Y_t = \frac{\Delta G-b_1\Delta T}{1-b_1-b_0 l}$	$\Delta GD_t = (\Delta G-\Delta T) - \frac{b_0(\Delta G-b_1\Delta T)}{1-b_1-b_0 l}$

Source: Reduced form equations for MS_t, Y_t and GD_t and Appendix.

In case 2, such money supply expansion has feed-back effects in the goods market since as it increases, the value of wealth has increased (here, NW = MS+K), while in case 1 it does not or does very slightly, since NW is defined as NW = MS+K+GD, and the increased MS is offset by decreased GD under this latter case.[82]

Therefore, since the income multiplier is greater in case 2 than in case 1 provided $\Delta G = \Delta T$, the change in money supply (which equals $b_o Y$) is greater under case 2 and hence the change in government debt is not as large,[83] but if ΔG is greater than ΔT, it is not possible to generalize and state that the change in money supply is greater under case 2.

Finally, we must mention that the relative values of $b_o 1$ and $1-b_1$ play an important but different role under either case 1 or 2. In case 1, they determine the way in which increased deficits are financed (i.e., the distribution between money supply and government debt); in case 2, they determine the possibility for the system to attain a stable equilibrium after G-T has increased[84] : if $b_o 1$ is greater than

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82. See the different cases for $b_o\ 1/[1-b_1)$ in last part of case 1.

83. This has been proven for case $\Delta G = \Delta T$ in general, and for the case $\Delta G > \Delta T$ with a numerical example since the general proof is more difficult in this latter case. See technical appendix at end of chapter.

or equal to $1-b_1$, the deficit will cause continuous and increased changes in the money supply and on income (each increase is larger than the previous one). Hence, there is no stable equilibrium for these endogenous variables. (See Table 3-14.)

2. Policy Procedure:Money Supply Instrument

Determination of the Instrument Value

This is determined by estimating the interest rate ($\tilde{r}$) from the IS and substituting its value into the LM. The result is:

$$M^* = \frac{1}{c_1'+c_1+c_o l}[(c_o(1-b_1+b_o)Y^*+c_o b_1 T- c_o G-c_o a_1-c_o a_2-c_o lK]$$

Implications of this Procedure

The reduced form of the interest rate that becomes the endogenous,clearing variable under the present policy is:

$$r = \frac{1}{b_o(c_1'+c_1)+c_o(1-b_1)}[(1-b_1-lb_o)MS^*-$$

$$b_o(a_1+a_2-b_1T+G+lk)-(1-b_1)U_{md}-b_oU_i-b_oU_c]$$

If we compare this reduced form with that in case 1, there are only slight differences, such as with respect to the effect of a change in G and T on the value of r. Here, government debt affects interest

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84. In this case 2, they determine the stability condition for any exogenous variable appearing in the reduced form for income

Table 3-14. Model A-5. Instrument Interest Rate. Comparision of Cases 1 and 2: Implications of Relative Values of $b_0 l$ and $(1-b_1)$;

	Case 1	Case 2
$\frac{b_0 l}{1-b_1} = 1$	$\Delta MS > \Delta G-\Delta T$ $\Delta GD < 0$	No stable system. (no equilibrium)
$\frac{b_0 l}{1-b_1} < 1$	$\Delta MS \leqq \Delta-\Delta T$ $\Delta GD \geqq 0$	Stable system
$\frac{b_0 l}{1-b_1} > 1$	$\Delta MS >> \Delta G-\Delta T$ $\Delta GD << 0$	No stable system. (no equilibrium)

rates only through G and T but not through consumption as in case 1. However, the effect of disturbances is the same in both cases.

Reduced Form of Income

It is given by:

$$Y = \frac{1}{(c_1'+c_1)b_o+c_o(1-b_1)}[(c_1'+c_1+c_o l)MS^*+c_o(a_1+a_2)$$

$$-c_o b_1 T+c_o G+c_o lK+c_o(U_i+U_c)-(c_1'+c_1)U_{md}]$$

Again, the only differnce with case 1 lies in the effect of changes in G and T on income, and this is due only to the definition of Net Worth.

In this case, only current budget conditions affect income, and not past period ones as happened in case 1 above. Furthermore, even the effect of current budget is different here than it was in case 1; under case 2 this effects are:

$$\Delta Y = \nu c_o(\Delta G_t-b_1\Delta T_t)$$

while under case 1 it was:

$$\Delta Y_t = \nu(c_o(\Delta G_t-b_1\Delta T_t)+c_o l(\Delta G_t-\Delta T_t))$$

These expressions show that a balanced budget ($\Delta G = \Delta T$) has the same impact on current income under either case; but an unbalanced budget which would include the increasing deficit situation (i.e., $\Delta G>\Delta T$) will have a greater impact on current income under case1 than

Table 3-15. Model A-5: Effects of ΔG and ΔT on Current Income, Money Supply and Interest Rates under Alternative Policies and Cases.

Policy	Model A-5 Case	Effect on Y (ΔY_t)	Effect on M (ΔM_t)	Effect on r (Δr_t)
Interest rate policy	Case 1	$\frac{1}{1-b1}\left(1(\Delta G-\Delta T)+\Delta G-b_1\Delta T \right)$	$\left(\frac{b0}{1-b_1} \right)\left(1(\Delta G-\Delta T)+\Delta G-b_1\Delta T \right)$	--
	Case 2	$\left(\frac{1}{1-b_1-b_0 1} \right)\left(\Delta G-b_1\Delta T \right)$	$\left[\frac{b_0}{1-b_1-b_0 1} \right] \Delta G-b_1\delta T)$	--
Money supply policy	Case 1	$\nu c_0\left[1(\Delta G-\Delta T)+\Delta G-b_1\Delta T\right]$ where $\nu = \frac{1}{b_0(c\|+c_1)+c_0(1-b_1)}$	--	$-\nu b_0[1(\Delta G-\Delta T)+\Delta-b_1\Delta T]$
	Case 2	$\nu c_0(\Delta G-b_1\Delta T)$	--	$-\nu b_0(\Delta G-b_1\Delta T)$

Source: Reduced form equations in text.

Table 3-16. Model A-5: Comparison of the Effect of ΔG and ΔT on Income under Alternative Monetary Policies (r^*) and MS^*) and under Alternative Cases 1 and 2.

A) Alternative policies, same case		
Case → Budget changes ↓	Case 1 (NW=MS+GD+K)	Case 2 (NW=MS+K)
$\Delta G = \Delta T$	$\Delta Y_{r*} > \Delta Y_{MS*}$	$\Delta Y_{r*} > \Delta Y_{MS*}$
$\Delta G > \Delta T$	$\Delta Y_{r*} > \Delta Y_{MS*}$	$\Delta Y_{r*} > \Delta Y_{MS*}$
B) Alternative cases, same policy		
Policies → Budget changes ↓	Interest rate policy	Money supply policy
$\Delta G = \Delta T$	$\Delta Y_2 > \Delta Y_1$	$\Delta Y_2 = \Delta Y_1$
$\Delta G > \Delta T$	uncertain result	$\Delta Y_1 > \Delta Y_2$

NOTE: Δ_{r*} = change in Y under an interest rate policy.

ΔY_1 = change in Y under Case 1.

Source: Table 3-15.

under case 2.

A final wrap-up from Model A-5 would be that, under case 1 only, past deficits affect current income and the monetary variables that become endogenous given the monetary policy procedure being followed.

On the other hand, under either case 1 or 2, an unbalanced or a balanced budget will affect current income but this effect will be stronger under an interest rate policy than under a money supply policy. (See Table 3-15) However, if the interest rate policy is followed, such effect on current income is stronger under case 2 than under case 1, given that $\Delta G = \Delta T$. While if a money supply policy is followed, the effect on Y is the same under both case if $\Delta G = \Delta T$ and stronger in case 1 if $\Delta G > \Delta T$. See Table 3-16.

With respect to the effect on the monetary variable that becomes endogenous under the policy under effect, we cannot definitively state under which policy the budget conditions will have a greater effect. All we can state in this respect is that under an interest rate policy the effects on the money supply will be larger in case 2 if $\Delta G = \Delta T$, while if $\Delta G > \Delta T$, the conclusion is uncertain, depending on the value of the parameters involved in the multiplier and in the magnitude of the respective ΔG and ΔT. See Table 3-17.

Under a money supply policy the effects on interest rates are the same in cases 1 and 2 if $\Delta G = \Delta T$ but are greater under case 1 if $\Delta G > \Delta T$.

Table 3-17. Model A-5: Comparison of the Effect of ΔG and ΔT on the Endogenous Monetary Variables under Alternative Monetary Policies (r* and MS*) and under Alternative Cases 1 and 2.

	A) Alternative policies, same case	
Cases → / Budget change ↓	Case 1	Case 2
$\Delta G = \Delta T$	$\Delta M_{r*} \gtrless \Delta r_{M*}$ uncertain	$\Delta M_{r*} \gtrless \Delta r_{M*}$ uncertain
$\Delta G > \Delta T$	uncertain	uncertain
	B) Alternative cases, same policy	
	Interest rate policy	Money supply policy
$\Delta G = \Delta T$	$\Delta M_2 > \Delta M_1$	$\Delta r_2 = \Delta r_1$
$\Delta G > \Delta T$	$\Delta M_2 \gtrless \Delta M_1$ uncertain	$\Delta r_1 > \Delta r_2$

NOTE: ΔM_{r*} = change in M under an interest rate policy.
ΔM_2 = change in M under case 2.
Source: Table 3-15

II. ANALYTICAL SIMULATION OF SHOCKS

A. Summary of Results: Models A-1 to A-5

In this section we proceed to give an overall summary of the results obtained under each economic structure and under each monetary policy procedure. Table 3-18 presents such results, showing for each of the specified models, and for each policy procedure, the reduced form equation for income and for the resulting endogenous monetary variable.

As we can see, whenever the interest rate policy is followed, the money supply becomes the endogenous monetary variable and therefore its reduced form equation is shown. On the contrary, when a money supply policy is implemented it is the interest rate that becomes endogenous and hence its reduced form equation becomes the one of interest.

On the other hand, in Model A-2 under the money supply policy we find two cases: one in which the instrument is money supply itself and the other where the the instrument is another monetary aggregate that is not sensitive to interest rates and hence is easier to control.

Table 3-18. Summary of Results.

Model numbers	Policy procedure (instrument used)	Value of instrument	Reduced Form Equations for Income and monetary endogenous variables
A-1 (original)	r^*	$r^* = \frac{Y^*-\alpha}{\beta}$	$MS = (c_0+b_0\beta)r^* + b_0\alpha+b_0U_{is}+U_{md}$
		$\beta = \frac{c_1}{1-b_1} < 0 \quad \alpha = (G+A_1+A_2/1-b_1$	$Y = \alpha + \beta r^* + U_{is} \quad U_{is} = \frac{U_c+U_i}{1-b_1}$
	MS^*	$MS^* = b_0Y^* + c_0\hat{r}$	$r = \frac{1}{b_0\beta + c_0} (MS^* - b_0\alpha - b_0U_{is} - U_{md})$
		where $\hat{r} = \frac{Y^*-\alpha}{\beta}$	
		$U_{is} = \frac{U_c + U_i}{1-b_1}$	$Y + \frac{1}{b_0\beta+c_0} [\beta MS^* + c_0\alpha+c_0U_{is} - \beta U_{md}]$

Table 3-18. (continued)

Model numbers	Policy procedure (instrument used)	Value of instrument	Reduced Form Equations for Income and monetary endogenous variables
A-2 (MS interest elastic)	r^*	$r^* = \frac{Y^* - \alpha}{\beta}$	$MS = (c_0+b_0\beta)r^* + b_0\alpha + b_0U_{is} + U_{md}$ $Y = \alpha + \beta r^* + U_{is} \quad U_{is} = \frac{U_i}{U_i}$
	Money supply Case A	$A_3^* = b_0Y^* + (c_0-c_3)\left(\frac{Y^* - \alpha}{\beta}\right)$ $\beta = \frac{c_1}{1-b_1}$ $U_{is} = \frac{U_c+U_i}{1-b_1}$	$MS = A_3^* + c_3r + U_{ms}$ $r = \frac{1}{b_0\beta-c_3+c_0} (A_3^*-b_0\alpha+U_{ms}-U_{md}-b_0U_{is})$ $Y = \frac{1}{b_0\beta-c_3+c_0} [\beta A_3^*-(C_3-c_0)\alpha-(c_3-c_0)U_{is} + \beta(U_{ms}-U_{md})$
	Case B	$MS^* = b_0Y^* + c_0\hat{r}$ $U_{is} = \frac{U_c+U_i}{1-b_1}$	$A_3 = b_0Y^* + c_0r - c_3r - U_{ms}$ $r = \frac{1}{c_0+b_0\beta} (MS^*-b_0\alpha-U_{md}-b_0U_{is})$ $Y = \frac{1}{c_0+b_0\beta} [\beta MS^*+c_0\alpha+c_0U_{is}-\beta U_{md}]$
Model numbers	Policy procedure (instrument used)	Value of instrument	Reduced Form Equations for Income and monetary endogenous variables
A-3 (Investment interest-inelastic)	r^*	r^* determined by past information on r that is considered good market conditions.	$MS = \frac{b_0}{1-b_1} [A_1+I+\bar{G}+U_c+U_i] +c_0r^*+U_{md}$ $Y = \frac{1}{1-b_1} (A_1+I+\bar{G}+U_c+U_i)$
	MS^*	MS^* determined by past experience.	$r = \frac{-b_0}{c_0(1-b_1)} (A_1+I+G+U_c+U_i) + \frac{MS^*}{c_0} - \frac{U_{md}}{c_0}$ $Y = \frac{1}{1-b_1} (A_1+I+G+U_c+U_i)$

Table 3-18. (continued)

Model
A-4

Reduced Form Equations (income and other endogenous variables)

Policy procedure (instrument used)	Value of instrument	Reduced Form Equations for Income and endogenous monetary variables
r^*	$r^* = \frac{Y^* - \alpha + \beta_2 \tilde{P} e}{\beta_2}$	$MS = (c_0 + b_0\beta_2)r^* + b_0\alpha - b_0\beta_2 \dot{P}\, e + b_0 U_{is} + U_{md}$ $Y = \alpha + \beta_2 r^* - \beta_2 \dot{P}\, e + U_{is}$ $U_{is} = \frac{U_c + U_i}{1 - b_1}$ $\beta_2 = \frac{\beta_1}{1 - \dot{P}\, e} = \frac{c_1}{(1-b_1)(1-\dot{P}\, e)}$
	Case A $\dot{P}\, e + \sum_{i=1}^{n} \left(\frac{Y_{t-1} - Y_{-t-1}}{Y_{t-i-1}} \right)$	Revision $(\tilde{P}e_{ij} = \dot{P}\, e_{ij})$ $\quad Y_{ij} = Y^*_{ij} + U_{is_{ij}}$ $(\tilde{P}e_{ij} \neq \dot{P}\, e_{ij})$ $\quad Y_{ij} = y^*_{ij} + \beta_2(\tilde{P}e_{ij} - \dot{P}\, e_{ij}) + U_{is_{ij}}$ where $\tilde{P}e_{ij} - \dot{P}\, e_{ij}$ = random
		No revision (rule) $(\tilde{P}e_{ij} \neq \dot{P}\, e_{ij})$ $\quad Y_{ij} = Y^*_{ij} + \beta_2(\tilde{P}e_{ij} - \dot{P}\, e_{ij}) + U_{is_{ij}}$ where: $Y_{ij} = Y^*_{ij} + u_{is_{ij}} + \beta_2 \left(-A_{ij} \frac{y_{i1} - Y_{i-1,4}}{Y_{i-1,4}}\right.$ $A_{i,2} \frac{Y_{i,2} - Y_{i,1}}{Y_{i,1}} - \ldots - A_{ij} \frac{Y_{ij-1} - Y_{i,j-2}}{Y_{i,j-2}}$ for $j>1$
	Case B $\dot{P}\, e = \sum_{i=1}^{n} \left(\frac{(M_{t-i} - M_{t-i-1})}{M_{t-i-1}} \right)$	Revision(discretion) $(\tilde{P}e_{ij} = \dot{P}\, e_{ij})$ $\quad Y_{ij} = Y^*_{ij} + U_{is_{ij}}$ $\tilde{P}e_{ij} \neq \dot{P}\, e_{ij})$ $\quad Y_{ij} = Y^*_{ij} + \beta_2(\tilde{P}e_{ij} - \dot{P}\, e_{ij}) + U_{is_{ij}}$ where $\tilde{P}e_{ij} - \dot{P}\, e_{ij}$ = random
		No revision (rule) $(\tilde{P}e_{ij} \neq \dot{P}\, e_{ij})$ $\quad Y_{ij}\ Y^*_{ij} + \beta_2(\tilde{P}e_{ij} - \dot{P}\, e_{ij}) + U_{is_{ij}}$ Where: $Y_{ij} = Y^*_{ij} + \beta_2 \left(-W_i \frac{M_{i1} - M_{i-1,4})}{M_{i-1,4}} \right)$ $-W_{i2} \frac{(M_{i2} - M_{i1})}{U_{is_{ij}}} \ldots W_{ij} \frac{M_{ij-1} - M_{i,j-1}}{M_{i,j-2}}$

Table 3-18. (continued)

	Model A-4	
MS*	$MS^* = Y^*(b_0 + \frac{c_0}{\beta_2}) - \frac{c_0\alpha}{\beta_2} + c_0\bar{\dot{P}}e$	$r = \frac{1}{b_0\beta_2 c_0}[MS^* - b_0\alpha + b_0\beta_2\dot{P}e - U_{md} - b_0U_{is}]$ $Y = \frac{1}{b_0\beta_2 + c_0}(\beta_2 MS^* + c_0\alpha - \beta_2 c_0\dot{P}e + c_0U_{is} - \beta_2 U_{md})$
	Case A $\dot{P}e = \sum_{i-1}^{h}(\frac{Y_{t-1} - Y_{t-i-1}}{Y_{t-i-1}})A_i$	Revision (discretion) $(\bar{\dot{P}}e_{ij} = \dot{P}e_{ij})$ $Y_{ij} = Y^*_{ij} + (\frac{1}{b_0\beta_2 + c_0})c_0U_{is_{ij}} - \beta_2U_{md_{ij}}$ $(\bar{\dot{P}}e_{ij} \neq \dot{P}e_{ij})$ $Y_{ij} = Y^*_{ij} + (\frac{1}{b_0\beta_2 + c_0})(c_0\beta_2(\bar{\dot{P}}e_{ij} - \dot{P}e_{ij}) + c_0U_{is_{ij}} - \beta_2U_{md_{ij}})$ where $(\bar{\dot{P}}e_{ij} - \dot{P}e_{ij})$ is random.
		No Revision(rule) $(\bar{\dot{P}}e_{ij} \neq \dot{P}e_{ij})$ $Y_{ij} = Y^*_{ij} + \frac{A_{ij}}{b_0\beta_2 + c_0}$ $\frac{c_0\beta_2}{b_0\beta_2 + c_0}[-A_{i1}(\frac{Y_{i1} - Y_{i-1,4}}{Y_{i-1,4}}) - A_{i2}(\frac{Y_{i2} - Y_{i,1}}{Y_{i,1}}) \ldots A_{ij}(\frac{Y_{ij-1} - Y_{i,j-2}}{Y_{i,j-2}})]$ where $A_{ij} = c_0U_{is_{ij}} - \beta_2U_{md_{ij}}$
	Case B	Revision $(\bar{\dot{P}}e_{ij} = \dot{P}e_{ij})$ $Y_{ij} = Y^*_{ij} + (\frac{1}{b_0\beta_2 c_0})(c_0U_{is_{ij}} - \beta_2U_{md_{ij}})$ $(\bar{\dot{P}}e_{ij} \neq \dot{P}e_{ij})$ $Y_{ij} = Y^*_{ij} + \frac{1}{b_0\beta_2 + c_0})(c_0\beta_2(\bar{\dot{P}}e_{ij} = \dot{P}e_{ij}) + c_0U_{is_{ij}} - \beta U_{md_{ij}})$ where $(\bar{\dot{P}}e_{ij} - \dot{P}e_{ij})$ is random.
		No revisioon $(\bar{\dot{P}}e_{ij} \neq \dot{P}e_{ij})$ $Y_{ij} = Y^*_{ij} + \frac{A_{ij}}{b_0\beta_2 c_0}$ where: $A_{ij} = c_0U_{is_{ij}} - \beta_2U_{md_{ij}}$

Table 3-18. (continued)

Model numbers	Policy procedure (instrument used)	Value of instrument	Reduced Form Equations for Income and monetary endogenous variables
A-5 (wealth effect) Case 1	r^*	$R^* = \frac{1}{c'_1+c_1}[Y^*(1-b_1)-A_1-A_2 + T(1+b_1)-G(1+l) -lMS_{t-1} - lGD_{t-1}-lK]$	$MS = c_0r^*+b_0[\frac{A_1+A_2}{1-b_1} + \frac{l}{1-b_1}(G_t-T_t) + \frac{G_t-b_1T_t}{1-b_1} + \frac{l}{1-b_1}\sum_{i=1}^{n}(G_{t-i}-T_{t-i}) + \frac{l}{1-b_1}GD_{t-n-1} + \frac{lMS_{t-n-1}}{1-b_1} + r^*)\frac{c'_1+c_1}{1-b_1}) + \frac{U_i+U_c}{1-b_1}$ $Y = \frac{1}{1-b_1}[A_1+A_2+l(G_t-T_t)+(G_t-b_1T_t) + l\sum_{i-1}^{n}(G_t-T_{t-1}) + lGD_{t-n-1} + lMS_{t-n-1} + r^*(c'_1+c_1) + U_i+U_c]$
	MS^*	$MS^* = \frac{c_0}{C'_1+C_1}[Y^*(1-b_1) -A_1-A_2+T(l+b_1) -G(1+l)-LMS_{t-1} - lGD_{t-1}-lK] + b_0Y^*$	$r = \frac{1}{c'_1b_0+c_1b_1+c_0(1-b_1)}[(1-b_1)MS^*-b_0(A_1+A_2)+b_0(l+b_1)T-Gb_0(1+l) -b_0lMS^*-b_0lGD_{t-1}-b_0lK-b_0(U_c+U_i) - (1-b_1)U_{md}]$ $Y = \frac{1}{b_0(c'_1+c_1)+(c_0(1-b_1)}[c'_1+c_1+c_0l)MS^* + c_0(A_1+A_2)-c_0(l+b_1)T + c_0(1+l)G+c_0lGD_{t-1}+c_0lK+c_0U_i + c_0U_c = (c'_1+c_1)U_{md}]$
Case 2	r^*	$r^* = \frac{1}{c'_1+c_1+l}[Y^*(1-b_1-lb_0) -A_1-A_2+b_1T -G - lK]$	$MS = \frac{1}{1-b_1-b_0l}[(c_0-c_0b_1+b_0c'_1+b_0c_1)r^*+b_0(A_1+A_2+G-b_1T +lK)+b_0(U_i+U_c)+(1-b_1)U_{md}]$ $Y = \frac{1}{1-b_1-b_0l}[A_1+A_2+G-b_1T+lK+(c'_1+c_1+lc_0)r^* + U_c + U_i + lU_{md}]$
	MS^*	$MS^* = \frac{1}{c'_1+c_1+lc_0}[(c_0(1-b_1)+b_0)Y^* + c_0b_1T-c_0G-c_0A_1 - c_0A_2-c_0lK]$	$r = \frac{1}{b_0(c'_1+c_1)+c_0(1-b_1)}[(1-b_1-lb_0)MS^* - b_0(A_1+A_2-b_1T+G+lK) - (1-b_1)U_{md}-b_0U_i-b_0U_c]$ $Y = \frac{1}{(c'_1+c_1)b_0+c_0(1-b_1}[(c'_1+c_1+c_0l)MS^*+c_0(A_1+A_2) -c_0b_1T+c_0G+c_0lK+c_0(U_i+U_c)-(c'_1+c_1)U_{md}]$

Under the latter approach both interest rates and the money supply will be endogenous and therefore both of their reduced form equations are shown.

Model A-3, where investment has been transformed into an interest inelastic function, turns out to be a case in which monetary policy cannot be directed towards attaining Y^*.

Model A-4, with the introduction of price expectations, allows for the possibility of dynamic effects. Therefore, when analyzing the sensitivity of the endogenous variables to different shocks, we will also need to refer to the dynamic multipliers. The price expectation mechanism, for which two different assumptions were considered, is the key element in the determination of these dynamic effects. On the other hand, the "revision" and "non revision" cases lead to different results with respect to these dynamic effects depending upon the policy procedure and the price expectations mechanism assumed.

Finally Model A-5 considers the wealth effect in Consumption, and our analysis shows how society's attitude with respect to government debt (i.e.,whether the debt is viewed as an asset or as a deferred liability) will render different implications with respect to the effect of shocks in the economy under alternative monetary policies, and also with respect to the effect of increasing deficits under alternative policies.

B. Effect of Each Type of Shock, Same Procedure but Different Models.

1. Interest Rate Policy

Interest Rate Policy: Effect of Shocks on Income

A general finding is that in models A-1, A-2 and A-3, under an interest rate policy, only real sector's shocks can cause deviations of income from target.

Then we have model A-4 which shows that under an interest rate target, income would be affected by real sector's shocks but also by changes in price expectations. Two cases are considered: case A, that assumes that price expectations are a function of past income growth, and case B, where price expectations depend upon past money growth. For both of these cases we introduce the possibilities that in the short run (period j) monetary authorities either revise or not their estimation of public's expectations. If they do revise and their revision coincides with the true price expectations the change in price expectations, has no effect in income since the authorities have revised r^* to be consistent with the new price expectations in achieving the target income. On the other hand, if their revised price expectations do not coincide with the true ones, there will be a deviation of Income from target, but this deviation will be random.

Finally, when there is no revision process in the short run, if there is a change in price expectations, income will deviate from target, but this deviation will not be random but will depend upon the price expectation formation mechanism. In case A it will depend upon past income changes, which are affected by past real sector's shocks. In case B, it depends upon past money supply changes which are affected by past shocks in the real and monetary sectors, as becomes evident from the money supply reduced form equation.

Therefore, under an interest rate policy procedure we have a situation in which both real and monetary shocks will cause income to deviate from target: current income is affected by current real sector shocks and by past monetary sector shocks through the price expectations variable. This will become clearer and more specific in the next section where we obtain the impact and dynamic multipliers for each type of shock, under each model and each policy procedure.

We then have model A-5, where the wealth effect in consumption has been introduced. Here we also studied two alternative cases, which differ only with respect to the definition of the non-human wealth variable that we called "net worth". In case 1, net worth was defined as the sum of government debt, money supply and the capital stock, while in case 2 we excluded government debt from the definition. It turns out that under case 1, when an interest rate policy is used only real sector shocks affect income as is seen from its reduced form equation. However, in case 2 we also find monetary shocks affecting income changes. This arises because a monetary shock, by causing money supply to be accommodative, will affect the real sector by changing net worth, which affects consumption, hence causing income to change (i.e., an LM shift causes an IS shift). Under case 1, the money supply also changed but it caused government debt to change in the opposite way leaving the value of net worth unchanged as a result of the monetary shocks.

In sum we have two situations in which, under an interest rate policy, both the real and monetary shocks will cause equilibrium income to change and hence to deviate from target if the economy starts at such target level. They are:

1. When investment is affected by price expectations, these being determined by past monetary growth, and assuming monetary authorities do not revise their estimates of society's price expectations in the short run, but only whey they set the instrument value every long run period.
2. When the consumption function is affected by non-human wealth or net worth, the latter defined as money supply plus stock of capital only; that is, government debt is not viewed by society as part of their net worth.

Interest Rate Policy: Effects of Shocks on Money Supply.

Now, since the money supply is the endogenous monetary variable under an interest rate policy, we proceed to analyze its reduced form equation under the alternative models we have.

Table 3-18 shows that both real and monetary shocks will cause the money supply to change under all of the models studied. It is also a general result that the greater the income sensitivity of the demand for money (i.e., b_0), the greater will be the effects of real sector's shocks on the money supply. On the other hand, the marginal propensity to consume (hereinafter MPC) also plays a role under all models where the effects of real sector shocks on the money supply will be greater the higher the MPC is. However we must note that under model A-4, in the no-revision cases real sector shocks affect money supply both directly and indirectly through the price expectation variable; this leads to the result that both current and past real sector shocks affect current changes in the money supply (See multipliers of the different shocks in Table 3-19).

Finally, under model A-5 case 2, the effect of these shocks on money supply is larger due to the larger multiplier that results because the money supply affects consumption through net worth.

We can conclude that the effect of real sector shocks on the money supply are greater under these circumstances:

1. An economy where investment depends upon price expectations, which are themselves dependent upon past money growth rates or upon past income growth rates.

Table 3-19. Model Summary: Impact Effect of Real Sector Shocks on Income under a Money Supply Policy ($\Delta Y_t/\Delta UC_t$)

Model:	Effect on income $\Delta Y_t/\Delta U_e = \Delta Y_t/\Delta U_i$	Rank according to magnitude of effect*
A-1	$\frac{c_0}{(1-b_1)(b_0\beta+c_0)} = \frac{c_0}{b_0c_1+c_0(1-b_1)}$	3
A-2 (both cases)	$\frac{c_0}{b_0c_1+c_0(1-b_1)}$	3
A-3	$\frac{1}{1-b_1}$	1
A-4	$\frac{c_0(1-\dot{P}\ e)}{b_0c_1+c_0(1-b_1)}$	2
A-5	$\frac{c_0}{b_0(c_1+c_1)+c_0(1-b_1)}$	4

* In descending order, for example, rank 1 is largest effect.

Source: Table 3-18.

2. An economy where consumption is affected by the non-human wealth component defined by the money supply and the stock of capital.

Monetary sector shocks are also present in the money supply reduced form equation under all models. In models A-1, A-2, A-3 and A-4 case A, their role is exactly the same: a one to one correspondence between the shock and the money supply change (i.e., the impact multiplier equals 1). However, under model A-4 with the assumption that price expectations are determined by past money growth and that there is no revision, current money supply will change due to both current and past monetary shocks, with the latter affecting the money supply through changes in price expectations.

Finally, under model A-5, when net worth is defined as money supply plus the capital stock (case 2), the effect of monetary shocks on the money supply is greater than under the case in which net worth has been defined differently (case 1), a result that is due to the different roles played by the money supply in the real sector when the definition of net worth is changed. In sum, if an interest rate policy is pursued, monetary shocks will cause greater money supply variability under economic structures that have these characteristics:

1. Investment is affected by price expectations which themselves depend upon past money growth, and the monetary authorities do not revise their price expectations estimates in the short

run.

2. Net worth, which is part of the public's wealth concept, does not include government debt considerations. That is, government debt is viewed as future burden on the tax system, or postponement of tax payments rather that current wealth, and therefore is not included in the public's wealth variable.

2. Money Supply Policy

Money Supply Policy:Effects of Shocks on Income.

Under a money supply policy, as Table 3-18 shows, the reduced form of income includes both real and monetary shocks in models A-1, A-2, A-4 and A-5. Under model A-3, given the non-sensitivity of investment to interest rates, income is determined solely by the real sector and therefore only real sector shocks can cause income to change. Then if we analyze each type of shock individually we observe that under model A-3 the effect of real sector shocks on income is given by the full real sector's multiplier (i.e., $\frac{1}{1-b_1}$).

We can see that the effect under model A-4 is greater than under A-1 and A-2, due to the effect of price expectations. Finally, under model A-5 the effect of real sector shocks on income is smaller than in A-1, A-2 and A-4, due to the larger interest sensitivity of

Table 3-20. Model Summary: Impact Effect of Monetary Shocks on Income under a Money Supply Policy ($\Delta Y/\Delta U_{ma}$)

Model	Effect (impact multipliers) $\Delta Y/\Delta U_{md}$
A-1	$\frac{-\beta}{b_0\beta+c_0} = \frac{-c_1}{b_0c_1+c_0(1-b_1)}$
A-2	$\frac{-c_1}{b_0c_1+c_0(1-b_1)}$
A-3	0
A-4	$\frac{-c_1}{b_0c_1+c_0(1-b_1)(1-\beta e)}$
A-5	$\frac{-(c'_1+c_1)}{b_0(c'_1+C_1)+c_0(1-b_1)}$

Source: Table 3-18.

spending ($c_1+c'_1$ as compared to c_1 before). (See Table 3-19).

With respect to monetary shocks we see in Table 3-20 that their effect on income is the same under A-1 and A-2, and greater in model A-5(due to c_1').[85]

Comparing models A-1 and A-2 with model A-4 it is obvious that the multiplier is lower in the latter due to the element "1 + Pe" in the denominator. However, under case A current income will also be affected by past monetary shocks and hence we would have a dynamic multiplier greater than the impact multiplier. We will see this in a section below.

Under the money supply policy it is possible to rank each model according to the degree of sensitivity of income to monetary and real sector shocks (impact multipliers); ranking them from the one in which there is less sensitivity to the one with greater sensitivity we obtain:

Monetary Shocks

A-3, A-4, A-1 and A-2, A-5.

Real Sector Shocks

A-5, A-1 and A-2, A-4, A-3

85. See proof in Appendix.

As a conclusion it seems that under a money supply policy monetary shocks have no effect under a structure where investment (and consumption) is insensitive to interest rates. Then, among the rest of the structures under analysis, the lowest impact is found in the case in which investment is affected by price expectations, although the dynamic effect under case A is greater than the effect of such impact. The largest impact of monetary shocks on income is under model A-5 and this is due to the larger interest-sensitivity of the real sector. On the other hand, under model A-5, real sector shocks have the smallest effect on Y, while under model A-3 these effects are maximized. Finally, comparing Tables 3-19 and 3-20 we conclude that, with the exception of model A-3, if the interest sensitivity of spending in the real sector is larger than the interest sensitivity of the demand for money, the effects of monetary sector shocks on income will be greater than the effects of real sector shocks, under a money supply policy. Also, the larger the (positive) price expectations, the greater the effects of real sector shocks and the lower the effects of monetary shocks on income.

Money supply Policy: Effect of Shocks on Interest Rates.

We now proceed to compare the effects of different shocks on interest rates under the money supply policy. From Tables 3-21 and 3-22, it is evident that interest rates are affected by both real sector and monetary sector shocks.

Table 3-21. Model Summary: Impact and Dynamic Effects of Monetary Sector Shocks on Interest Rates under a Money Supply Policy

Model	Impact effect of a shock on the demand for money (U_{md})	Impact effect of a shock on the money supply (U_{ms})	Dynamic effect of U_{md}
A-1	$\frac{-(1-b_1)}{b_0c_1+c_0(1-b_1)}$	not applicable	--
A-2 case A	$\frac{-1}{b_0\beta-c_3+c_0}$	$\frac{-1}{b_0\beta-c_3+C_0}$	--
A-2 case B	$\frac{-1}{c_0+b_0\beta}$	not applicable	--
A-3	$\frac{-1}{c_0}$	not applicable	--
A-4 case A	$\frac{-1}{b_0\beta_2+c_0}$	not applicable	if no revision: $\frac{b_0\beta_2{}^2A_i}{(b_0\beta_2+c_0)^2Y_{ij-1}}+\ldots+\ldots\frac{(-1)^kb_0c_0{}^{k-1}\beta_2A_{ij}\ldots A_{ij+k}}{(b_0\beta_2+c_0)^{k+1}Y_{i,j-k+1}\ldots Y_{i,j}}$
A-4 case B	$\frac{-1}{b_0\beta_2+c_0}$	not applicable	zero
A-5	$\frac{-(1-b_1)}{b_0(c'_1+c_1)+c_0(1-b_1)} > 0$	not applicable	--

Source: Appendix.

Table 3-22. Model Summary: Effect of Real Sector Shocks (U_i' and U_c) on Interest Rates under a Money Supply Policy.

Model	Impact Multiplier	Dynamic Multipler
A-1	$\frac{-b_0}{b_0c_1+c_0(1-b_1)}$	--
A-2 case A	$\frac{-b_0}{b_0c_1+(c_0-c_3)(1-b_1)}$	--
case B	$\frac{-b_0}{b_0c_1+c_0(1-b_1)}$	--
A-3	$\frac{-b_0}{c_0(1-b_1)}$	--
A-4	$\frac{-b_0}{b_0c_1+c_0(1-b_1)}$	$\frac{1}{1-b_1}\left[\frac{b_0c_0\beta_2A_{ij}}{(b_0\beta_2+c_0)^2Y_{ij-1}} - \ldots + \frac{(-1)^{k-1}c_0^k\ b_0\beta_2^k\ A_{ij}\cdots A_{ij+k-1}}{(b_0\beta_2+c_0)^{k+1}Y_{ij-1}\cdots Y_{ij+k-2}}\right.$ k = # short-run periods ellapsed after shock.
case B	$\frac{-b_0}{b_0c_1+c_0(1-b_1)}$	--
A-5	$\frac{-b_0}{b_0(C'_1+c_1)+c_0(1-b_1)}$	

Source: Technical notes, chapter 3.